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THE CELL METHOD

A Purely Algebraic Computational Method in Physics and Engineering

BY **ELENA FERRETTI**

The Cell Method (CM) is a computational tool that maintains critical multidimensional attributes of physical phenomena in analysis. This information is neglected in the differential formulations of the classical approaches of finite element, boundary element, finite volume, and finite difference analysis, often leading to numerical instabilities and spurious results.

This book highlights the central theoretical concepts of the CM that preserve a more accurate and precise representation of the geometric and topological features of variables for practical problem solving. Important applications occur in fields such as electromagnetics, electrodynamics, solid mechanics, and fluids. CM addresses non-locality in continuum mechanics, an especially important circumstance in modeling heterogeneous materials. Professional engineers and scientists, as well as graduate students, are offered:

- A general overview of physics and its mathematical descriptions;
- Guidance on how to build direct, discrete formulations;
- Coverage of the governing equations of the CM, including non-locality;
- Explanations of the use of Tonti diagrams; and
- References for further reading.

ABOUT THE AUTHOR

Elena Ferretti is an Assistant Professor in the School of Engineering and Architecture, Alma Mater Studiorum — University of Bologna, Italy. Her research focuses on the development of numerical codes for fracture mechanics in brittle heterogeneous materials, based on the relationship between non-local approaches and discrete formulations. She is the author or coauthor of numerous conference and journal publications and is the recipient of the 2009 ICCES / S. Ramanujan Young Investigator Award in Computation.

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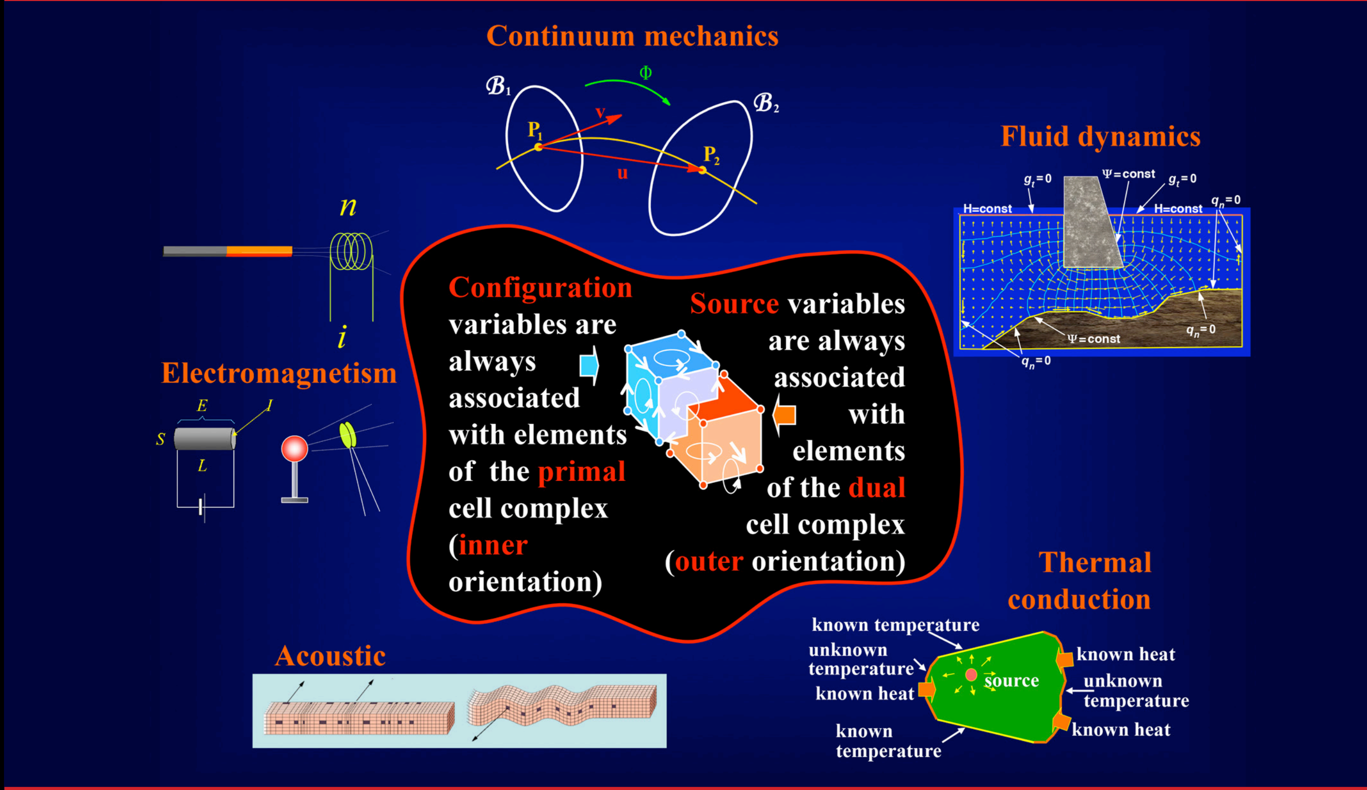

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Elena Ferretti
Bologna, October 2013

Preface

The computational methods currently used in physics are based on the discretization of the differential formulation, by using one of the many methods of discretization, such as the finite element method (FEM), the boundary element method (BEM), the finite volume method (FVM), the finite difference method (FDM), and so forth. Infinitesimal analysis has without doubt played a major role in the mathematical treatment of physics in the past, and will continue to do so in the future, but, as discussed in Chapter 1, we must also be aware that several important aspects of the phenomenon being described, such as its geometrical and topological features, remain hidden, in using the differential formulation. This is a consequence not of performing the limit, in itself, but rather of the numerical technique used for finding the limit. In Chapter 1, we analyze and compare the two most known techniques, the iterative technique and the application of the Cancelation Rule for limits. It is shown how the first technique, leading to the approximate solution of the algebraic formulation, preserves information on the trend of the function in the neighborhood of the estimation point, while the second technique, leading to the exact solution of the differential formulation, does not. Under the topological point of view, this means that the algebraic formulation preserves information on the length scales associated with the solution, while the differential formulation does not. On the basis of this observation, it is also proposed to consider that the limit provided by the Cancelation Rule for limits is exact only in the broad sense (i.e., the numerical sense), and not in the narrow sense (involving also topological information). Moreover, applying the limit process introduces some limitations as regularity conditions must be imposed on the field variables. These regularity conditions, in particular those concerning differentiability, are the price we pay for using a formalism that is both very advanced and easy to manipulate.

The Cancelation Rule for limits leads to point-wise field variables, while the iterative procedure leads to global variables (Section 1.2), which, being associated with elements provided with an extent, are set functions (Section 1.3). The use of global variables instead of field variables allows us to obtain a purely algebraic approach to physical laws (Chapter 4, Chapter 5), called the direct algebraic formulation. The term "direct" emphasizes that this formulation is not induced by the differential formulation, as is the case for the so-called discrete formulations that are often compared to it (Section 1.4). By performing densities and rates of the global variables, it is then always possible to obtain the differential formulation from the direct algebraic formulation.

Since the algebraic formulation is developed before the differential formulation, and not vice-versa, the direct algebraic formulation cannot use the tools of the differential formulation for describing physical variables and equations. Therefore, the need for new suitable tools arises, which allows us to translate physical notions into mathematical notions through the intermediation of topology and geometry. The most convenient mathematical setting where to formulate a geometrical approach of physics is algebraic topology, the branch of the mathematics that develops notions corresponding to those of the differential formulations, but based on global variables instead of field variables. This approach leads us to use algebra instead of differential calculus. In order to provide a better understanding of what using algebra instead of differential calculus means, Chapter 2 deals with exterior algebra (Section 2.1) and geometric algebra (Section 2.2), the two fundamental settings for the geometric study of spaces not just of geometric vectors, but of other vector-like objects such as vector fields or functions. Algebraic topology and its features are then treated in Chapter 3.

The Cell Method (CM) is the computational method based on the direct algebraic formulation developed by Enzo Tonti.[1] Tonti's first papers on the direct algebraic formulation date back to 1974 (see Reference Section). The main motivation of these early works is that physical integral variables are naturally associated with geometrical elements in space (points, lines, surfaces, and volumes) and time elements (time instants and time intervals), an observation that also allows us to answer the question on why analogies exist between different physical theories:

> "*Since in every physical theory there are integral variables associated with space and time elements it follows that there is a correspondence between the quantities and the equations of two physical theories in which the homologous quantities are those associated with the same space-time elements.*"

The CM was implemented starting from the late '90s. The first theory described by means of the direct algebraic formulation was electromagnetism in 1995, followed by solid mechanics and fluids.

The strength of the CM is that of associating any physical variable with the geometrical and topological features (Chapter 1, Chapter 4), usually neglected by the differential formulation. This goal is achieved by abandoning the habit to discretize the differential equations. The governing equations are derived in algebraic manner directly, by means of the global variables, leading to a numerical method that is not simply a new numerical method among many others. The CM offers an interdisciplinary approach, which can be applied to the various branches of classical and relativistic physics. Moreover, giving an algebraic system of physical laws is not only a mathematical expedient, needed in computational physics because computers can only use a finite number of algebraic operators. The truly algebraic formulation also provides us with a numerical analysis that is more adherent to the physical nature of the phenomenon under consideration (Chapter 1). Finally, differently from their variations, the global variables are always continuous through the interface of two different media and in presence of discontinuities of the domain or the sources of the problem (Section 1.2). Therefore, the CM can be usefully employed in problems with domains made of several materials, geometrical discontinuities (corners), and concentrated sources. It also allows an easy computation in contact problems.

Even if having shown the existence of a common mathematical structure underlying the various branches of physics is one of the most relevant key-points of the direct algebraic formulation, the purpose of this book is not that of explaining the origin of this common structure,

[1] **Enzo Tonti** (born October 30, 1935) is an Italian mathematical physicist, now emeritus professor at the University of Trieste (Italy). He began his own scientific activity in 1962, working in the field of Mathematical Physics, the development of mathematical methods for application to problems in physics.

as already extensively done by Tonti, in his publications. Our focus will be above all on giving the mathematical foundations of the CM, and highlighting some theoretical features of the CM, not yet taken into account or adequately discussed previously. To this aim, the basics of the CM will be exposed in this book only to the extent necessary to the understanding of the reader.

One of the contributions given in this book to the understanding of the CM theoretical foundations is having emphasized that the Cancelation Rule for limits acts on the actual solution of a physical problem as a projection operator, as we have already pointed out. In Section 1.1.3, this new interpretation of the Cancelation Rule for limits is discussed in the light of the findings of non-standard calculus, the modern application of infinitesimals, in the sense of non-standard analysis, to differential and integral calculus. It is concluded that the direct algebraic approach can be viewed as the algebraic version of non-standard calculus. In fact, the extension of the real numbers with the hyperreal numbers, which is on the basis of non-standard analysis, is equivalent to providing the space of reals with a supplementary structure of infinitesimal lengths. In other words, it is an attempt to recover the loss of length scales due to the use of the Cancelation Rule for limits, in differential formulation. For the same reasons, the CM can be viewed as the numerical algebraic version of those numerical methods that incorporate some length scales in their formulations. This incorporation is usually done, explicitly or implicitly, in order to avoid numerical instabilities. Since the CM does not need to recover the length scales, because the metric notions are preserved at each level of the direct algebraic formulation, the CM is a powerful numerical instrument that can be used to avoid some typical spurious solutions of the differential formulation. The problem of the numerical instabilities is treated in Chapter 6, with special reference to electromagnetics, electrodynamics, and continuum mechanics. Particular emphasis is devoted to the associated topic of non-locality in continuum mechanics, where the classical local continuum concept is not adequate for modeling heterogeneous materials in the context of the classical differential formulation, causing the ill-posedness of boundary value problems with strain-softening constitutive models. Further possible uses of the CM for the numerical stability in other physical theories are under study, at the moment.

Some other differences and improvements, with respect to the papers and books on the CM by other Authors, include:

- The CM is viewed as a geometric algebra, which is an enrichment (or more precisely, a quantization) of the exterior algebra (Section 2.2.1). Since the geometric algebra provides compact and intuitive descriptions in many areas, including quantum mechanics, it is argued (Section 4.1) that the CM can be used even for applications to problems of quantum mechanics, a field not yet explored, at the moment.
- The p-space elements and their inner and outer orientations are derived inductively, and not deductively. They are obtained from the outer product of the geometric algebra and the features of p-vectors (Section 2.2.2). It is shown that it is possible to establish an isomorphism between the orthogonal complement and the dual vector space of any subset of vectors, which extends to the orientations. Some similarities with the general Banach spaces are also highlighted. It is concluded that the notions of inner and outer orientations are implicit in geometric algebra.
- Each cell of a plane cell complex is viewed as a two-dimensional space, where the points of the cell, with their labeling and inner orientation, play the role of a basis scalar, the edges of the cell, with their labeling and inner orientation, play the role of basis vectors, and the cell itself, with its inner orientation, plays the role of basis bivector (Section 3.5).

- Space and time global variables are treated in a unified four-dimensional space/time cell complex, whose elementary cell is the tesseract (Sections 3.8, 5.1.2-5.1.4). The resulting approach shows several similarities with the four-dimensional Minkowski spacetime. Moreover, the association between the geometrical elements of the tesseract and the "space" and "time" global variables allows us to provide an explanation (Section 4.4) of why the possible combinations between oriented space and oriented time elements are in number of 32, as observed by Tonti and summarized in Section 4.1. It is also shown how the coboundary process on the discrete p-forms, which is the tool for building the topological equations in the CM, generalizes the spacetime gradient in spacetime algebra (Section 5.1.2).
- The configuration variables with their topological equations, on the one hand, and the source variables with their topological equations, on the other hand, are viewed as a bialgebra and its dual algebra (Section 4.1). This new point of view allows us to give an explanation of why the configuration variables are associated with space elements endowed with a kind of orientation and the source variables are associated with space elements endowed with the other kind orientation.
- The properties of the boundary and coboundary operators are used in order to find the algebraic form of the virtual work theorem (Section 4.2).
- It is made a distinction between the three coboundary operators, $\delta^{\mathbf{D}}$, $\delta^{\mathbf{C}}$, and $\delta^{\mathbf{G}}$, which, being tensors, are independent of the labeling, the three incidence matrices, **D**, **C**, and **G**, whose incidence numbers depend on the particular choice of labeling, and the three matrices, $\mathbf{T}^{\mathbf{D}}$, $\mathbf{T}^{\mathbf{C}}$, and $\mathbf{T}^{\mathbf{G}}$, which represent the coboundary operators for the given labeling of the cell complex (Section 5.1). In the special case where all the 1-cells of the three-dimensional cell complex are of unit length, all the 2-cells are of unit area, and all the 3-cells are of unit volume, $\mathbf{T}^{\mathbf{D}}$, $\mathbf{T}^{\mathbf{C}}$, and $\mathbf{T}^{\mathbf{G}}$ equal **D**, **C**, and **G**, respectively. If this is not the case, $\mathbf{T}^{\mathbf{D}}$, $\mathbf{T}^{\mathbf{C}}$, and $\mathbf{T}^{\mathbf{G}}$ are obtained with a procedure of expansion and assembling of local matrices, which is derived from the procedure of expansion and assembling of the stiffness matrix. The rows of **D**, **C**, and **G** give the right operators in the expansion step.
- Possible developments of the CM are investigated for the representation of reality through a purely algebraic unifying gravitational theory, theorized by Einstein during the last decades of his life (Section 6.4).

Elena
Bologna, October 2013

KEYWORDS

Cell method, heterogeneous materials, non-local models, non-standard analysis, bialgebra, Clifford algebra, discrete formulations, fracture mechanics, electromagnetics, electrodynamics, solid mechanics, fluid mechanics, space-time continuum, numerical instabilities, topological features of variables, graph theory, coboundary process, finite element method, boundary element method, finite volume method, finite difference method.

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