Effective Properties for Plain Concrete by Mono-Axially Compressed Cylinders

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Abstract
A new procedure is proposed for identifying mono-axial stress-strain relationship and Poisson ratio in compressed plain concrete. By considering the specimen as a structure, the procedure identifies effective properties from experimental data. This way of proceeding involves a modification of traditionally identified mono-axial stress-strain relationship and Poisson ratio. Results are presented for cylinders of various slendernesses.

Introduction
When identifying constitutive laws by experimental tests, the object in testing is never the material, as should be necessary, but a specimen, that is to say, a structure (Figure 1, [1]). On the base of partially analogous considerations, Rosati et. al. [2] proposed a complete response for concrete loaded in tension.

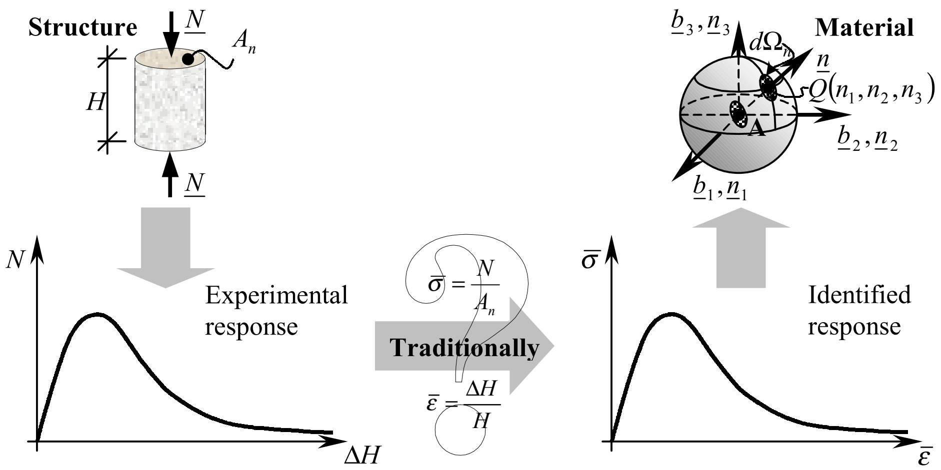

Figure 1. Traditional identification of mono-axial constitutive law by experimental tests.

Thus, experimental results characterise structural, and not material, behaviour. In this respect, the softening behaviour of the load-displacement curve, N-ΔH, for mono-axially compressed concrete has a meaning linked to structural instability.
In order to intend to derive the response of the material in compression, it is common practice to define the average stress $\bar{\sigma}$ and the average strain $\bar{\varepsilon}$ as shown in Figure 1. The meaning of constitutive law for monotonic strain processes is traditionally attributed to the $\bar{\sigma}$ - $\bar{\varepsilon}$ relationship. However, the following observations must be made:

1. The traditional identification procedure consists of a mere change of scale. Thus, experimental and identified curves are homothetic (Figure 1). Nevertheless, it is not possible to associate a physical meaning with the softening behaviour of a material response, as the concept of instability loses its sense in the infinitesimal neighbourhood of a point [3]. Besides, starting from the beginning of the 20th century, strain-softening has been widely regarded as inadmissible by several authors [4].
2. The $\bar{\sigma}$ - $\bar{\varepsilon}$ law is size-effect sensitive.

For all these reasons, it is right to hypothesise that the traditional identification procedure is not adequate enough to model the physical phenomenon in discussion. That is to say, to identify constitutive laws starting from experimental results it is necessary to evaluate all factors influencing a test result. Indeed, since the specimen is a structure, experimental results (*R*) depend not only on constitutive properties (*C*), but also on structural mechanics (*S*), on interactions between test-machine and specimen (*I*), and on test-machine metrological characteristics (*M*):

$$R = C + S + I + M \ .$$

It is then necessary to establish which are the scale factors to redefine a law in the σ - ε plane.

Identification approach of σ-ε effective behaviour in mono-axial compression

Named K_C, K_S, K_I, and K_M the weighed contributions assumed by *C*, *S*, *I*, and *M*, respectively, in the definition of *R*:

$$C = K_C R \ , \quad S = K_S R \ , \quad I = K_I R \ , \quad M = K_M R \ ;$$

it follows that $K_C + K_S + K_I + K_M = 1$.

All the contributions with the exception of the constitutive behaviour can be grouped in one factor $K = K_S + K_I + K_M$. With this position, the traditional identification between C and R, $C \equiv R$, is replaced by the relationship $C = (1 - K)R$.

This relationship allows evaluation of the constitutive properties, taking into account the behaviour of the specimen as a structure. Nevertheless, it is not of immediate use for identifying constitutive properties, since $K_C = K_C(R)$, $K_S = K_S(R)$, $K_I = K_I(R)$, and $K_M = K_M(R)$ are, generally speaking, load-step functions. That is to say, $K = K(R)$ is a load-step function, and not a constant of the performing test. In conclusion, as regards compressive tests on concrete, it is not possible to establish a homothetic correspondence between experimental load-displacement relationship and constitutive mono-axial stress-strain relationship. Moreover, since $K = K(R)$ is not of objective determination, one can only estimate it, in such a way as to identify an effective (and not constitutive) response, with regard to the material scale.

The main consequence of these considerations is the loss of the traditional identity between experimental and effective curve shape. In other words, effective curve may not exhibit the typical softening behaviour of experimental curve. Since it is impossible to associate a physical meaning with the strain-softening behaviour of a material response, it can be asserted that effective laws must be monotonically increasingly for any material.

Figure 2. Resistant structure at the end of the test.

An analysis of reciprocal ratios between K_C, K_S, K_I, and K_M for compressed concrete [3] showed that it is possible to assume $K \cong K_S$. Thus, to identify the scale factor of the σ axis with respect to the N axis (Figure 1), it is fundamental to introduce a parameter whose dimensions are those of an area and whose variation is linked to the structural mechanics. In the following, this parameter will be indicated as resistant area, A_{res}. In this hypothesis, any specimen can be viewed as being composed by a resistant structure (Figure 2), in which crack propagation never occurred, and a volume of incoherent material.

In this study, $A_{res} = A_n(1 - D)$ has been estimated in accordance with the Fracture Mechanics with Damage.

Also the effective stress $\sigma_{eff} = \overline{\sigma}\, A_n / A_{res}$, which is defined as the average stress acting on A_{res}, has a formulation derived from the Fracture Mechanics with Damage.

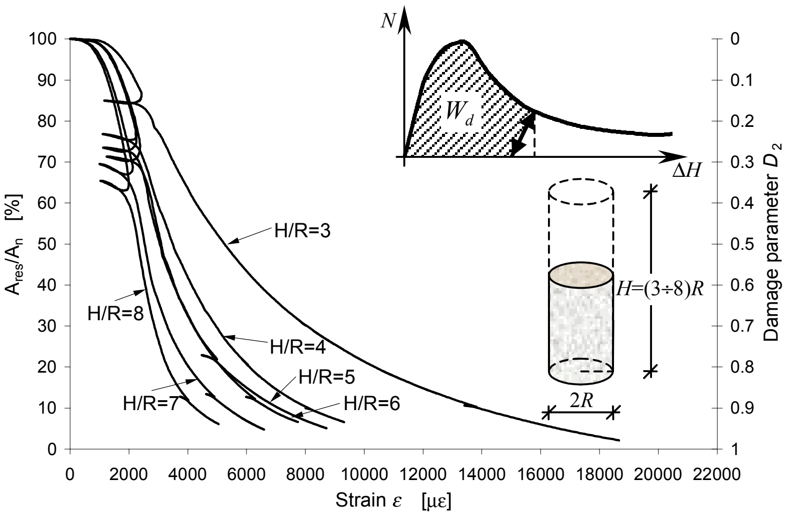

Figure 3. Evolution of resistant area and D_2 damage law for various slendernesses.

Nevertheless, the approach proposed in this study is very far from the manner of operation of Fracture Mechanics with Damage: in Fracture Mechanics with Damage, D is analytically formulated and it is considered as uniformly distributed on A_n; in this study, $D = D(R)$ is experimentally evaluated and it is considered localised in the volume of incoherent material. To evaluate $D = D(R)$, two experimental damage laws were employed in the case of concrete cylindrical specimens. The first damage law, $D_1 = 1 - V/V_0$ [5], where V_0 is the initial microseismic signal velocity, relates damage to the variation of microseismic signal velocity V. The second damage law, $D_2 = W_d/W_{d,t}$ [6], where $W_{d,t}$ is the total dissipated energy, relates damage to the dissipated energy W_d (Figure 3).

D_1 and D_2 turned out to be very close to each other [3].

Damage laws were experimentally derived for various specimen slendernesses. Figure 3 shows D_2 damage laws obtained for H/R ratios varying from 3 and 8. As can be seen in Figure 3, damage laws are size-effect sensitive. That is, the highest is the H/R ratio, the highest is D for every load-step.

As regard the scale factor of the ε axis with respect to the ΔH axis (Figure 1), the effective strain ε_{eff} has been identified considering that, in a generic unloading-reloading cycle, only conservative forces act. In this hypothesis, the instantaneous secant stiffness for the σ_{eff} - ε_{eff} law, $E_s = \tan\alpha$ (Figure 4), is represented by the slope of the unloading-reloading cycle for the current point. Thus, the point σ_{eff} - ε_{eff} is the intersecting point of the two lines $\sigma = \sigma_{eff}$ and $\sigma = E_s\varepsilon$.

The σ_{eff} - ε_{eff} relationships obtained for the 6 tested geometries fall inside the grey region in Figure 4. All these curves turned out to be increasingly monotonic and size-effect insensitive.

The average curve shape in the σ_{eff} - ε_{eff} plane is representative of the meso-scale material behaviour.

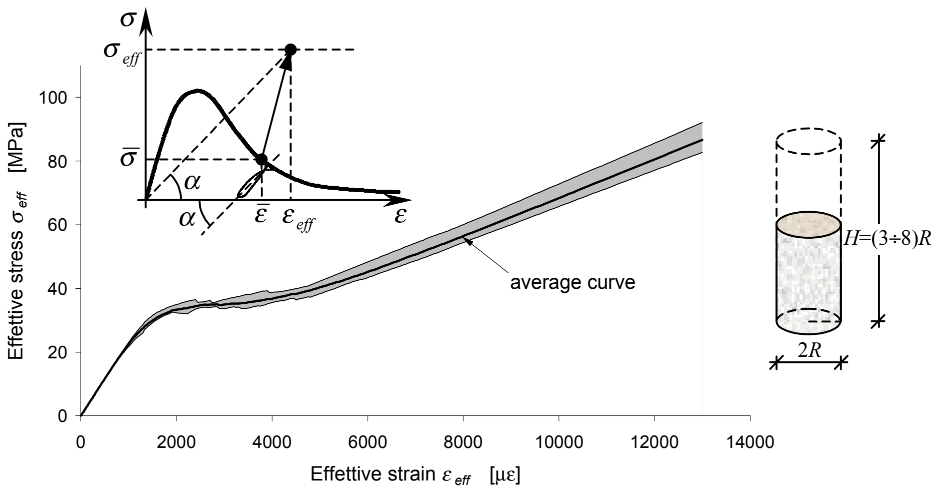

Figure 4. $\sigma_{eff}/\varepsilon_{eff}$ range for various slenderness and average curve.

Identification of Poisson ratio

With reference to the traditional approach, said $\varepsilon_r = \Delta R/R$ the radial strain and $\varepsilon_l = \Delta H/H$ the longitudinal strain for a mono-axial compressed solid, the Poisson ratio is defined as $\nu = -\varepsilon_r/\varepsilon_l$. While ε_l is of immediate determination, things are different for ε_r , since it is not easy to measure a radial strain. A way to solve this problem is to reduce the radial measure to a circumferential measure:

$$\varepsilon_r = \frac{\Delta R}{R} = \frac{2\pi\Delta R}{2\pi R} = \frac{\Delta crf}{crf}.$$

Operatively, radial strain can be acquired by means of a circumferential strain gauge [1, 3], maintained in the right position using a chain (Figure 5). Nevertheless, strain measurements acquired on cylindrical specimen surface are not employable to evaluate the Poisson ratio, because they are affected by crack openings. Even this time, the model traditionally assumed to identify a constitutive property is not accurate enough to interpret the physical problem. Plotting the resultant $\varepsilon_r/\varepsilon_l$ ratio in function of ε_l , you will find a monotonically increasing relationship [7] (Figure 5).
In order to obviate this problem, fibre optic sensors (FOSs) were utilised to acquire radial strains internally to the resistant structure (Figure 5). Plotting the $\varepsilon_r/\varepsilon_l$ ratio for this new acquisition, you will find that it is almost constant with ε_l (Figure 5).
Since in this study it was hypothesised that macro-cracks does not occur in the resistant structure, the new constant behaviour of $\varepsilon_r/\varepsilon_l$ could be considered more representative of the Poisson ratio ν than the traditional increasing behaviour of $\varepsilon_r/\varepsilon_l$ is. To evaluate the actual Poisson ratio, one must take into account the actual stress state in the resistant structure, since this state is tri-axial in any case.

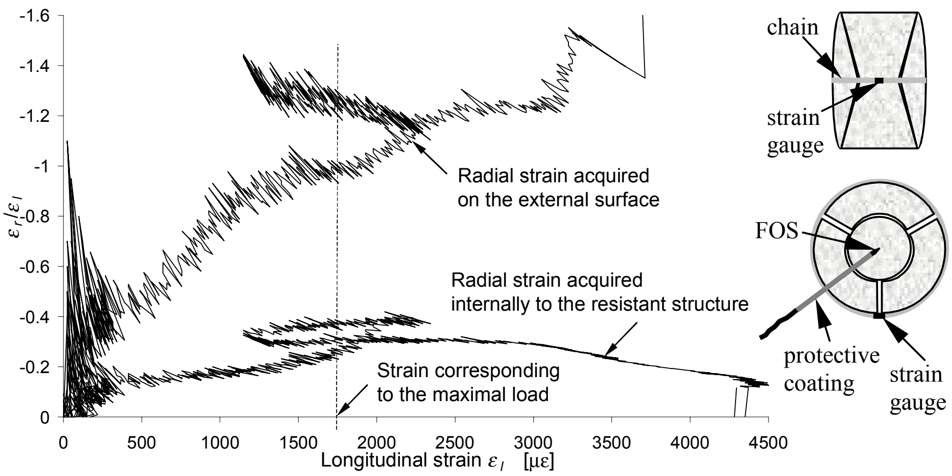

Figure 5. Traditional and new $\varepsilon_r/\varepsilon_l$ ratio.

Finally, new volumetric curve integrally belongs to negative field, while traditional volumetric curve also develops in positive filed [7]. Thus, on the contrary of what traditionally asserted [7], it is possible to assume that concrete never exhibits a dilatant behaviour.

Conclusions

Considering the specimen as a structure interacting with the test-machine, it was possible to identify a mono-axial σ_{eff} - ε_{eff} relationship for concrete [3], which is in good agreement with the requirement [3] of monotonically increasingly behaviour in the stress-strain plane. Moreover, this curve turns out to be size-effect insensitive. The proposed approach allows identification of a σ_{eff} - ε_{eff} relationship for the description of the meso-scale material behaviour. This relationship, together with adequate failure criteria, leads to structural analysis [3].
By means of strain acquisitions internally to the resistant structure, a very close Poisson ratio was estimated to be constant with the longitudinal strain [1]. From the constant value of the Poisson ratio, it follows that concrete never exhibits a dilatant behaviour [1]. What we know as dilatant behaviour of the concrete [7] comes from an erroneous acquisition of radial strain, which is affected by crack openings.
The qualitative evaluations of the Poisson ratio and of the specific variation of volume, from strain measurements acquired into the resistant structure, can provide useful information for the description of the effective response in tri-axial state.

References

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