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ON POISSON'S RATIO AND VOLUMETRIC STRAIN IN CONCRETE

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Abstract. A new procedure is proposed for identifying uniaxial stress-strain relationship and Poisson's ratio in compressed plain concrete. The procedure is based on the assumption of an internal core of intact material always present inside a specimen in uniaxial-compression. This involves a modification of the traditionally identified uniaxial stress-strain relationship and Poisson's ratio, which turns out to be almost independent of the loading step. The main finding concerns the volumetric strain, since it appears to be no real increase in the volume of a concrete solid when the solid is placed under pressure.

1. Introduction. As was shown in Ferretti (2001) and Ferretti (2004a), large reductions of the effective cross-sectional area, or resistant area (A_{res}), occur in a concrete specimen in a compression test. This is due to the propagation of macro-cracks through the specimen from the very beginning of the compression test. In cylindrical specimens, these cracks isolate an inner core of bi-conic shape (Fig. 2).

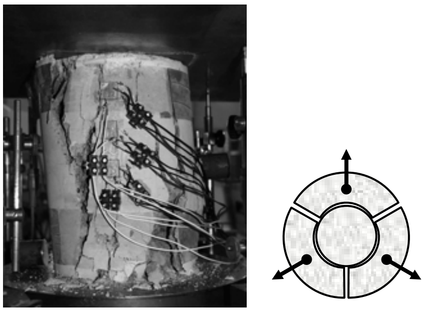
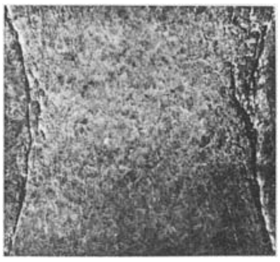


Figure 1. Concrete specimen at the end of the test, and scheme of slitting on the middle cross-section.

Figure 2. Concrete specimen at the end of the test, after removal of the outer part.

The outer part of the specimen, the one located around the inner core, is expulsed along the radial direction, as shown in the scheme in Fig. 1. This results in splitting of the outer part into several portions. The splitting takes place with propagation of sub-vertical macro-cracks, clearly visible on the external surface of the specimen (Fig. 1). Usually, the propagation of sub-vertical macro-cracks is considered as the actual failure mechanism in concrete, while the cause of these cracks, the propagation of bi-cone shaped macro-cracks, is not taken into account at all. In the present analysis, the sub-vertical cracks are considered as a secondary effect of the bi-conic failure mechanism. Due to the splitting, the outer part of the specimen looses its load-carrying capability. On the contrary, the inner core continues to carry load as the bi-conic cracks propagate. The final shape of the inner core, resulting from removal of the non-collaborating material, is shown in Fig. 2. No evident crack propagation seems to afflict this core. Thus, it represents the resistant structure at the end of the compression test. At intermediate steps of the load-process, we can observe a partial formation of the bi-conic inner core.

Splitting into inner core and outer part is observed also in compression tests on metamorphic rocks, like marble (Hudson et al., 1971; Fig. 3).



1:1 specimen;
4-inch diameter

1/3:1 specimen;
4-inch diameter

Figure 3. Longitudinal section of Georgia Cherokee marble specimens at an advanced state of failure (Hudson et al., 1971).

Since the propagation of bi-cone shaped crack does not characterize the final stage only, but progress with load from the very beginning of the test, during compression the resistant structure gradually evolves from the one coinciding with the specimen and the one in Fig. 2. This modification of resistant structure involves a modification of the resistant area, A_{res}. At an intermediate step of loading, A_{res} is slightly greater than the area of the minimal cross-section of the bi-conic core, due to the residual load-carrying capability of the outer part.

The effective stress σ_{eff} and the effective strain ε_{eff} have been introduced as constitutive parameters describing material behavior when the reduction of A_{res} is considered. An identifying procedure for deriving effective properties from experimental load-displacement (*N*-*u*) diagrams is presented by Ferretti (2001, 2004b). The identified σ_{eff} - ε_{eff} law turned out to be monotonic strictly non-decreasing, as expected from physical and mathematical considerations. In particular, Hudson et al. (1971) suggested that softening is not a material property, but is essentially due to scaling the applied force by the original cross-sectional area rather than the actual cross-sectional area (Fig. 4). Nevertheless, before Ferretti (2001) no one succeeded in tracking the effective cross-sectional area experimentally at each stage of the failure process, and an identification procedure for the effective law was lacking in literature. The effective law by Ferretti (2001), σ_{eff} - ε_{eff}, is also insensitive to size-effect, as a constitutive law should be.

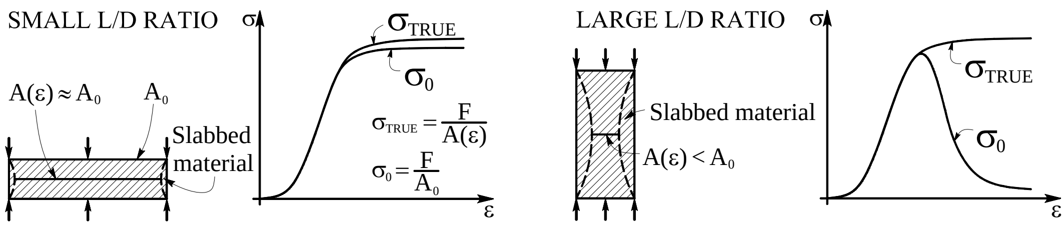


Figure 4. Effect of stress definition on the shape of the stress–strain curve ($\sigma_{TRUE}=\sigma_{eff}$; Hudson et al., 1971).

The σ_{eff} - ε_{eff} curve is representative of the meso-scale material behavior. This curve was used in a simulation program for modeling the compressive tests on six specimens with varying slenderness, L/D (Ferretti, 2001; Ferretti, 2005). The program is based on a Cell Method code for crack propagation analysis developed by Ferretti (2003). It makes it possible to upload the domain geometry as the dominant crack propagates. An intra-element propagation technique with automatic remeshing was used for this purpose. The code automatically computes the decrement of resistant area due to crack propagation (Ferretti, 2005). In agreement with the experimental results, the numerical *N*-*u* diagrams show softening, and the size-effect is well reproduced (Ferretti, 2005). Also, the *N*-*u* behavior in the tri-axial state of stress is well reproduced (Ferretti and Di Leo, 2003). The numerically predicted crack path is very close to the experimental crack path, for each L/D ratio (Ferretti, 2005).

Here, we discuss experimental acquisition of the radial strain. It will be shown how the acquisition is affected by inhomogeneous deformation caused by the experimental technique. One must be careful, therefore, when the Poisson's ratio and the volumetric curve are derived. A new technique is proposed for radial strain acquisition. Comparisons between the Poisson's ratio and volumetric curve derived from the traditional and the proposed technique are presented, showing how a non-accurate evaluation of the failure mechanism can lead to erroneous identification of the material properties.

2. Identification of the Poisson's ratio. With reference to the traditional approach, with $\varepsilon_r = \Delta R/R$ the radial strain and $\varepsilon_l = \Delta H/H$ the longitudinal strain for a uniaxially compressed solid, the Poisson's ratio is defined as $\nu = -\varepsilon_r/\varepsilon_l$. Whereas the value of ε_l is easy to acquire experimentally, things are different for ε_r , since it is not easy to measure a radial strain. One way to solve this problem is to use cylindrical specimens and reduce the radial measure to a circumferential measure of strain, ε_c . Actually, in cylinders ε_r and ε_c have the same value:

$$\varepsilon_r = \frac{\Delta R}{R} = \frac{2\pi\Delta R}{2\pi R} = \varepsilon_c . \qquad (1)$$

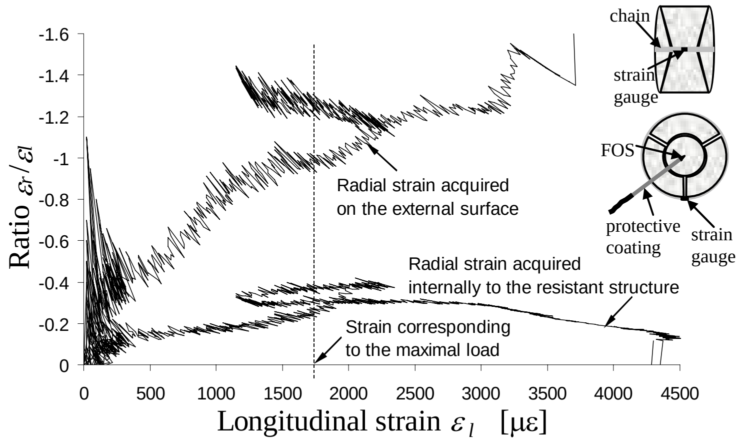


Figure 5. Ratios $\varepsilon_r/\varepsilon_l$: as conventionally defined and as presently proposed.

On the basis of Eq. (1), the radial strain can be acquired by means of a circumferential strain gauge. In Ferretti and Carli (1999) and Ferretti (2001), the circumferential strain gauge was maintained in the right position using a steel chain (Fig. 5). In the scheme in Fig. 5, the chain is positioned on the middle cross-section. In the following, whenever speaking of radial strains acquired on the external surface we will refer to values on this same section. The chain was not fixed on the specimen directly. It was closed on the specimen by means of two elastic springs, preventing the chain from dropping. Due to the high difference between the stiffness of the chain and of the two springs, during loading the chain does not deform and the relative displacement between the two ends is equal to $2\pi\Delta R$. Thus, a strain gauge positioned between the two ends of the chain (Fig. 5) measures the average circumferential strain, equal to the average radial strain.

It is worth noting that the equality between ε_r and ε_c in Eq. (1) is valid until the cracks start to propagate only. If macro-cracks propagate through the specimen, such as shown in Fig. 5, strain measurements acquired on cylindrical specimen surface are not real strains. They are affected by the crack openings (Fig. 5). Thus, they cannot be employed for evaluating the Poisson's ratio. The model traditionally assumed to identify the Poisson's ratio is not accurate enough to interpret the physical problem. The resultant $\varepsilon_r/\varepsilon_l$ ratio in function of ε_l (Fig. 5) is a monotonic nondecreasing relationship (Di Leo et al., 1979), rapidly reaching non-physical values. With the aim of verifying the assumed failure mechanism with opening of macro-cracks (Fig. 5), fiber optic sensors (FOSs) have been utilized (Ferretti, 2001) for acquiring radial strains into the presumed resistant structure (Fig. 5). The $\varepsilon_r/\varepsilon_l$ ratio for this new acquisition is almost independent of ε_l (Ferretti and Carli, 1999; Fig. 5). In the assumption that macro-cracks do not occur in the resistant core, the constant behavior of $\varepsilon_r/\varepsilon_l$ could be considered more representative of ν than the traditional increasing ratio $\varepsilon_r / \varepsilon_l$. To evaluate the actual ν, the actual stress state in the resistant structure must be taken into account, since this state is tri-axial in any case.

3. Identification of the volumetric strain. The volumetric strain ε_ϑ, the ratio of the change in the volume of a body that occurs when the body is placed under pressure, to the original volume of the body, is equal to the first invariant of strain, $I_{1\varepsilon}$, the trace of the strain tensor. In cylindrical specimens, the principal strains, ε_1, ε_2, and ε_3, are equal to ε_l, ε_r, and ε_c, with $\varepsilon_r = \varepsilon_c$ (Eq. (1)). Thus, in cylinders ε_ϑ is bonded to ε_l and ε_r as follows:

$$\varepsilon_\vartheta = I_{1\varepsilon} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \varepsilon_l + \varepsilon_r + \varepsilon_c = \varepsilon_l + 2\varepsilon_r . \quad (2)$$

ε_ϑ is considered positive if involving volume increase (dilatancy). If ε_r is acquired on the external surface, as the average ε_c, the volumetric curve, N-ε_ϑ, is, mostly, in the positive field (Brace et al., 1966; Di Leo et al., 1979; Fig. 6). When the confining pressure is very low, the dilatancy also arises in a wide variety of compressed rocks as the maximum stress becomes one-third to two-thirds the fracture stress at a given pressure (Brace et al., 1966), if ε_r is derived from the average ε_c [1]. On the contrary, the curve obtained in Ferretti and Carli (1999) by radial strain acquisitions into the presumed resistant structure of concrete specimens, by means of FOSs (Fig. 5), is in the negative field (Fig. 6).

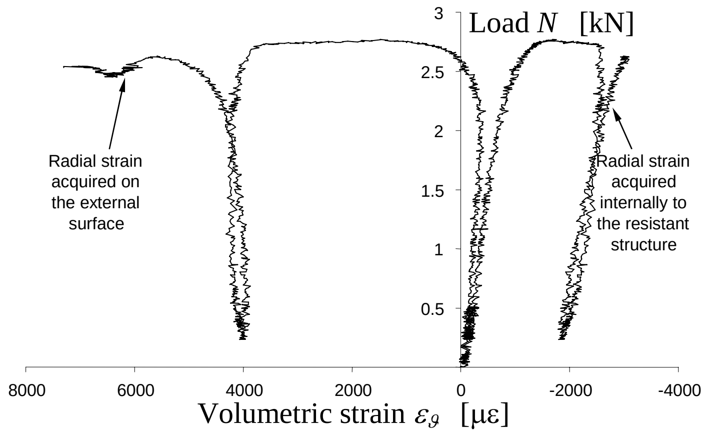


Figure 6. Traditional and identified volumetric curves for concrete specimens (Ferretti and Carli, 1999).

Since the radial strain derived from the average ε_c is affected by the crack openings, wile not the one acquired into the inner core (Fig. 5), even for the

[1] In Brace et al. (1966), circumferential gages extended to very nearly half of the full circumference were used in order to acquire the circumferential strain, assumed equal to the radial strain.

relationship N-ε_ϑ the curve following from FOS acquisitions is more representative of the actual concrete behavior than the traditional curve is. In this case, one can assert that concrete never exhibits dilatancy. Concrete dilatancy (Brace et al., 1966; Di Leo et al., 1979) is only an apparent effect, due to an experimental technique which inadequately evaluates the influence on acquired data of a failure mechanism with crack openings. It is worth noting that also in the study on rock dilatancy by Brace et al. (1966) was assumed that part of the observed dilatancy, particularly of brittle rocks, might be due to splitting. By analyzing the variation of the axial compressibility, the compressional wave velocity in axial direction, the circumferential compressibility, the compressional wave velocity in radial direction, and the volume changes in the granite, Brace et al. (1966) finally concluded that cracks form in the axial direction at a fraction of the maximum stress, leading to a volume increasing relative to elastic changes, in a first stage, and, then, to dilatancy. Thus, dilatancy apparently can be traced to open axial cracks (Brace et al., 1966). A discussion on the form which these cracks might have and the ways in which they might originate supports this conclusion (Brace et al., 1966). Now, strain acquisitions into the presumed resistant structure seem to confirm experimentally that dilatancy is actually an apparent effect, due to crack openings.

4. Conclusions. A concrete specimen under uniaxial monotonic compression in displacement-control test is characterised by a load (N)-displacement (u) diagram with softening. During the loading, the specimen exhibits a crack propagation pattern that depends on its structural nature and interaction with the test-machine. The N-u diagram itself is affected by the structural behavior and interaction between the specimen and the test-machine, since the crack propagation modifies the resistant structure.

Considering the specimen as a structure interacting with the test-machine, it was demonstrated in Ferretti (2004a) that the σ_{eff} - ε_{eff} curve exhibits a strictly positive derivative at the point corresponding to the peak of $\bar{\sigma}$ - $\bar{\varepsilon}$, the curve of the average stress versus the average strain. Moreover, a procedure to experimentally evaluate the law of the percentage decrease of resistant area has been proposed in Ferretti (2004b). This law led to monotone strictly nondecreasing and size-effect insensitive σ_{eff} - ε_{eff} curves.

By means of strain acquisitions into the presumed resistant structure, the Poisson's ratio was estimated to be almost independent of the longitudinal strain (Ferretti and Carli, 1999). From the constant value of the Poisson's ratio, it follows that concrete never exhibits dilatancy (Ferretti and Carli, 1999). What we know as concrete dilatancy (Brace et al., 1966; Di Leo et al., 1979) may come from erroneous acquisitions of radial strain, which are affected by crack openings.

The comparison between the Poisson's ratios resulting from acquisitions into the resistant core and on the external surface gives an indirect validation to the assumption of macro-cracks developing in concrete specimens during the load-carrying process. Indeed, the difference between the two moduli can be attributed to opening of macro-cracks. Moreover, the internally acquired modulus could not be constant if an internal core of intact material did not exist. Since the existence of an internal core of intact material is the basis of the σ_{eff} - ε_{eff} identification, these results also validate the procedure suggested by Ferretti (2004b).

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