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Crack Trajectory in Tensioned Concrete Plates using the Cell Method

E. Ferretti, E. Viola, A. Di Leo

Abstract

In this study, the problem of finding the complete trajectory of propagation in plates with internal straight cracks is extended to the non-linear field. In particular, results concerning concrete plates in bi-axial tensile loading are shown.

The concrete constitutive law adopted for this purpose is monotonically non-decreasing, as following according to previous studies of the Authors on monotonic mono-axial loading.

The analysis is performed in a discrete form, by means of the Cell Method (CM).

The aim of this study is both to test the new concrete constitutive law in biaxial tensile load and to verify the applicability of the CM in crack propagation problems for bodies of non-linear material.

The discrete analysis allows us to identify the crack initiation without using the stress intensity factors. Moreover, previous studies showed how the boundary conditions are no longer a problem with the CM. These two circumstances involve computational simplifications in cracked solids of finite dimensions. An example of computation in finite solids, the skew-symmetric four-point bending beam, is provided.

1 Introduction

For finding a crack trajectory, the variational principle of the most common crack theories has been used over the past three decades [9]. Criteria for the initiation of crack propagation can be obtained on the basis of both energy and force considerations. Historically, at first an energy fracture criterion was proposed by A. A. Griffith in 1920 [3] and G. R. Irwin formulated a force criterion in 1957 [5], while the same time demonstrating the equivalence of the two criteria. The Irwin force criterion for crack extension and the equivalent Griffith energy criterion completely solve the question of the limiting equilibrium state of a cracked continuous elastic body. Nevertheless, there exists a number of other formulations also establishing the limiting equilibrium state of a cracked body. Among these, the best known models are those of Leonov and Panasyuk (1959) [6], Dugdale (1960) [1], Wells (1961) [10], Novozhilov (1969) [8], and McClintock (1958) [7].

Usually, the variational problem of finding the limiting load and the correlated crack propagation direction is reduced to that of finding extreme points of a function of several variables [9]. In the present paper, the variational approach has been abandoned in favour of a discrete formulation of the crack propagation problem. The numerical calculation is then performed by a new numerical method for solving field equations: the Cell Method (CM) [10].

The use of a discrete formulation instead of a variational one is advantageous, since it does not require the definition of a model for treating the zone ahead of the crack edge. When studying crack problems for an elastic-perfectly plastic body with the energy equilibrium criterion, for instance, the solution is usually given in the case when the plastic deformation is concentrated in a narrow zone ahead of the crack edge. The thickness of this zone is of the order of elastic displacements. Moreover, when the plastic zone ahead of the edge is thin, the problem is reduced to the solution of an elastic problem instead of an elastic-plastic one. This reduction is based on the fact that, in the linearized formulation, a thin plastic zone may be schematically replaced by an additional cut along the face of which are applied forces replacing the action of the plastically deforming material. Attention is then drawn to the fact that the region of plastic non-linear effects in the model under consideration varies with the external load and represents a plastically deforming material in which the state of stress and strain must be determined from the solution of an elastic-plastic problem. With the discrete formulation, on the contrary, no hypothesis on the shape and dimensions of the plastic zone is needed, and the calculation is performed directly, without having to reduce the problem to an equivalent elastic one.
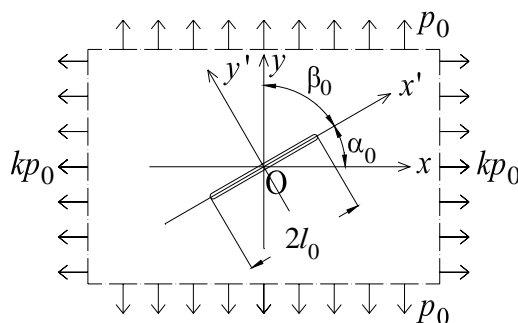

Figure 1: Load and geometrical set-up of the cracked plate.

The numerical code for crack trajectory analysis with the CM has been developed by E. Ferretti [2]. In this study, the code has been extended to provide results in the case of a concrete plate (Fig. 1)

tensioned at infinity by a load of intensity $p_x = kp_0$ parallel to the x-axis, and $p_y = p_0$ parallel to the y-axis. The plate has an initial straight crack of length $2l_0$ oriented at an angle α_0 to the x-axis (β_0 to the y-axis). The crack trajectory is provided for various values of k and α_0.

2 Crack extension criterion

The minimum load required to propagate a crack (limiting load) can be determined by using a variety of criteria:
- maximal normal stress criterion;
- maximal strain criterion;
- minimum strain energy density fracture criterion;
- maximal strain energy release rate criterion;
- damage law criteria.

In the present paper, the crack extension condition is studied in the Mohr-Coulomb plane. The limiting load is computed as the load satisfying the condition of tangency between the Mohr's circle and the Leon limit surface (Fig. 2).
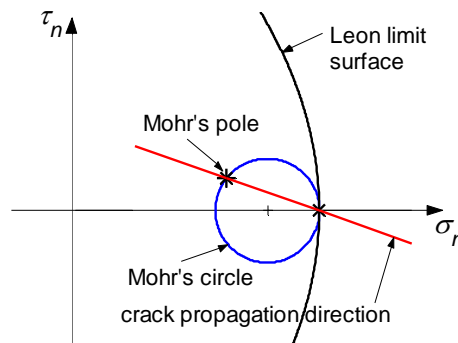

Figure 2: Leon limit surface in the Mohr-Coulomb plane.

Said c the cohesion, f_c the compressive strength, f_{tb} the tensile strength, τ_n and σ_n, respectively, the shear and normal stress on the attitude of external normal n, the Leon criterion in the Mohr-Coulomb plane is expressed as:

$$\tau_n^2 = \frac{c}{f_c}\left(\frac{f_{tb}}{f_c} + \sigma_n\right). \tag{1}$$

The Mohr's circle for the tip neighbourhood is identified by means of the physical significance associated with the CM domain discretization.

The CM divides the domain by means of two cell complexes, in such a way that every cell of the first cell complex, which is a simplicial complex, contains one, and one only, node of the other cell complex. In this study, a Delaunay/Voronoi mesh generator is used to generate the two meshes in two-dimensional domains. The primal mesh (the Delaunay mesh) is obtained by subdividing the domain into triangles, so that for each triangle of the triangulation the circumcircle of that triangle is empty of all other sites (Fig. 3). The dual mesh (the Voronoi mesh) is formed by the polygons whose vertexes are at the circumcenters of the primal mesh (Fig. 3). For each Voronoi site, every point in the region

around that site is closer to that site than to any of the other Voronoi sites. The conservation law is enforced on the dual polygon of every primal vertex.

To identify the Mohr's circle for the tip neighbourhood, an hexagonal element was inserted at the tip (Ferretti in press, Fig. 3). When the mesh generator is activated, the hexagonal element is divided into equilateral Delaunay triangles and a quasi-regular tip Voronoi cell is generated (the cell filled in gray in Fig. 3). This allow us to establish a correspondence between the tip stress field and the attitudes corresponding to the sites of the tip Voronoi cell. It has been shown [2] that the tension points correctly describe the Mohr's circle in the Mohr-Coulomb plane, for every rotation of the hexagonal element around the tip.
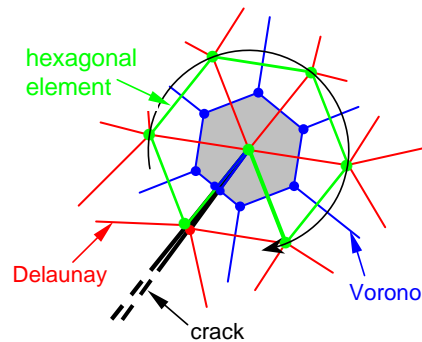

Figure 3: Hexagonal element for analysis in the Mohr-Coulomb plane.

The propagation direction is then derived as the direction of the line joining the tangent point to the Mohr's pole (Fig. 2).

3 Numerical results

Numerical results concerning the concrete plate loaded as shown in Fig. 1 are here presented. For symbols and conventions, refer to the same Fig. 1.

The concrete constitutive law adopted in this study is monotonically non-decreasing, in accordance with the identification procedure for concrete in mono-axial load provided in [3] (Fig. 4). It was shown [3] how this constitutive law turned out to be size insensitive for mono-axial compressive load. This result has made it possible to formulate a new concrete law in mono-axial loading, the effective law, which can be considered more representative of the material physical properties than the softening laws are. Now, the effective law is tested for applications in bi-axial tensile load.
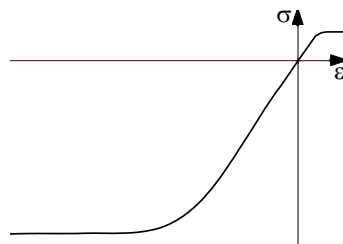

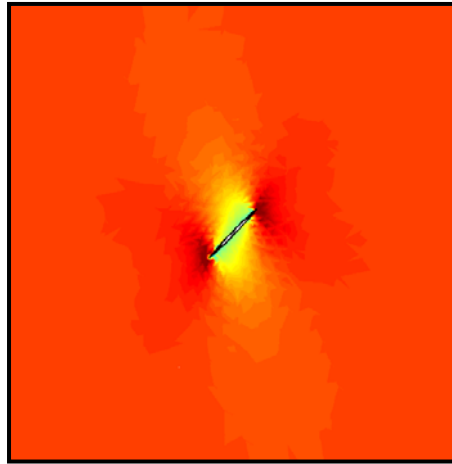Figure 4: Adopted constitutive law for concrete in mono-axial load.

Figure 5: 2D p_y analysis for the initial straight crack with $k = 0$, $\alpha_0 = \pi/4$.
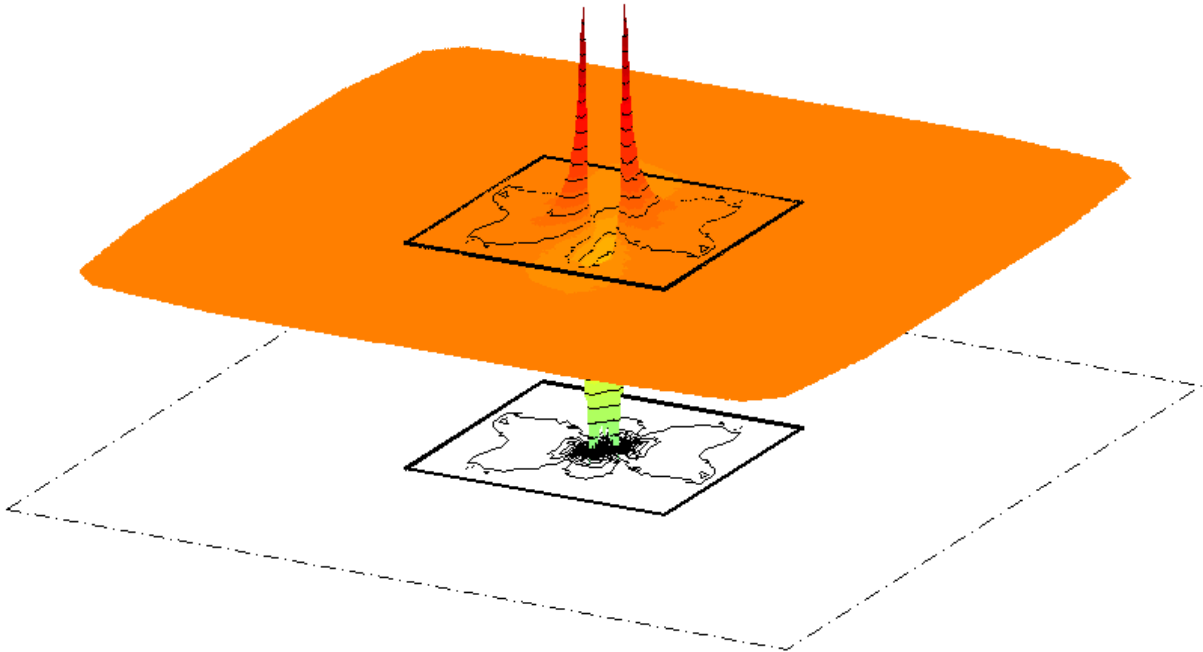

Figure 6: 3D p_y analysis and lines of equal p_y for the initial straight crack with $k = 0$, $\alpha_0 = \pi/4$.

The p_y analysis for $k = 0$ and $\alpha_0 = \pi/4$, plotted on the deformed configuration of a finite area around the crack, is shown in Fig. 5 for the initial straight crack. In this figure, the darker red color corresponds to the maximal tensile stress, while the darker green color corresponds to $p_y = 0$.

In Fig. 6, the p_y analysis is performed in 3D, with the level lines plotted in the plane $p_y = 0$. In this figure, the significance of colors is the same as in Fig. 5. The 3D plot correctly shows how p_y reaches a value numerically close to 0 in correspondence with the crack surface. The boxed area in the plane $p_y = 0$ of Fig. 6 is the area of Fig. 5. As all the level lines are internal to the boxed area, it can be assumed with good approximation that this area represents the stress extinction zone for the initial straight crack.

In Fig. 7 and Fig. 8, the 2D and 3D p_y analysis for $k = 0$ and $\alpha_0 = \pi/4$ is shown for a generic crack propagation step. The lighter colors of Figs. 7 and 8 with respect to those of Figs. 5 and 6 are representative of the progressive plate downloading corresponding to a crack propagation. For the same reason, the values in the third axis of Fig. 8 are smaller than those of Fig. 6. Moreover, in Fig. 7 it can be observed how a mono-axial compressive state of stress of small entity (cyan color) arises in the plate, due to the geometrical effect of the non-straight crack deformation in Mode I.

The crack trajectory is plotted in Fig. 9 for the value of the angle α_0 equal to $\pi/6$, $\pi/4$ and $\pi/3$, and for the factor k equal to 0, $1/4$, $1/2$, $3/4$ and 1. In this figure, also the two meshes of Delaunay-Voronoi were plotted for $k = 0$, in order to show how the adaptive mesh generator recreates the meshes for a generic crack propagation step (Delaunay mesh in red and Voronoi mesh in blue, as indicated in Fig. 1).
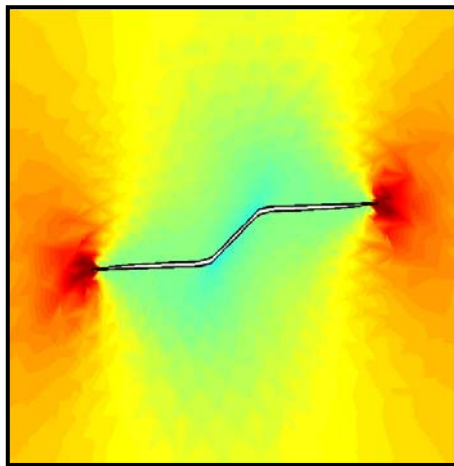

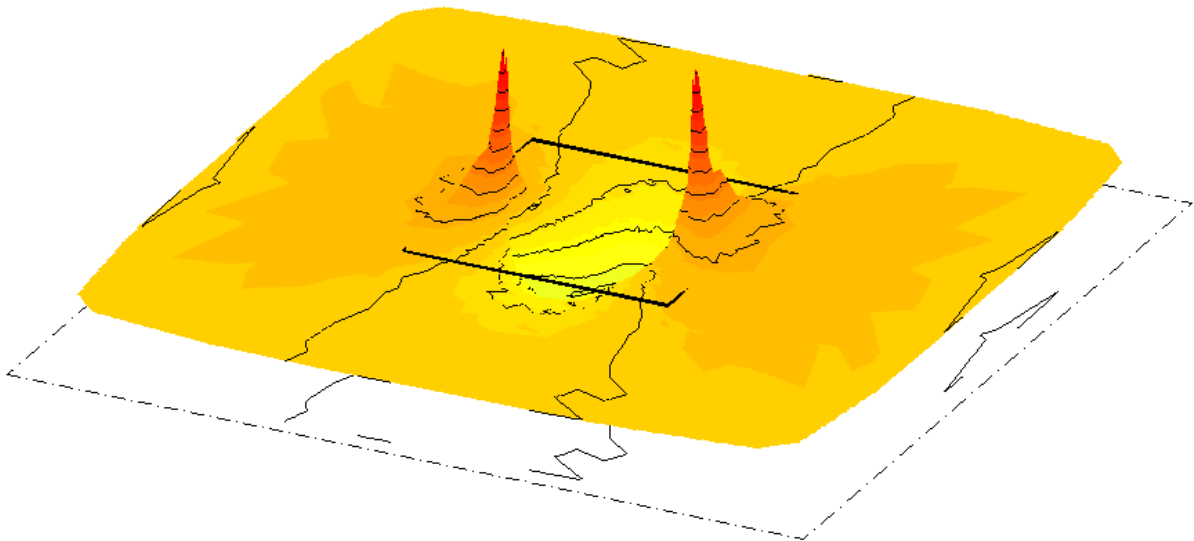Figure 7: 2D p_y analysis for a generic propagation step with $k = 0$, $\alpha_0 = \pi/4$.

Figure 8: 3D p_y analysis and lines of equal p_y for a generic propagation step with $k = 0$, $\alpha_0 = \pi/4$.
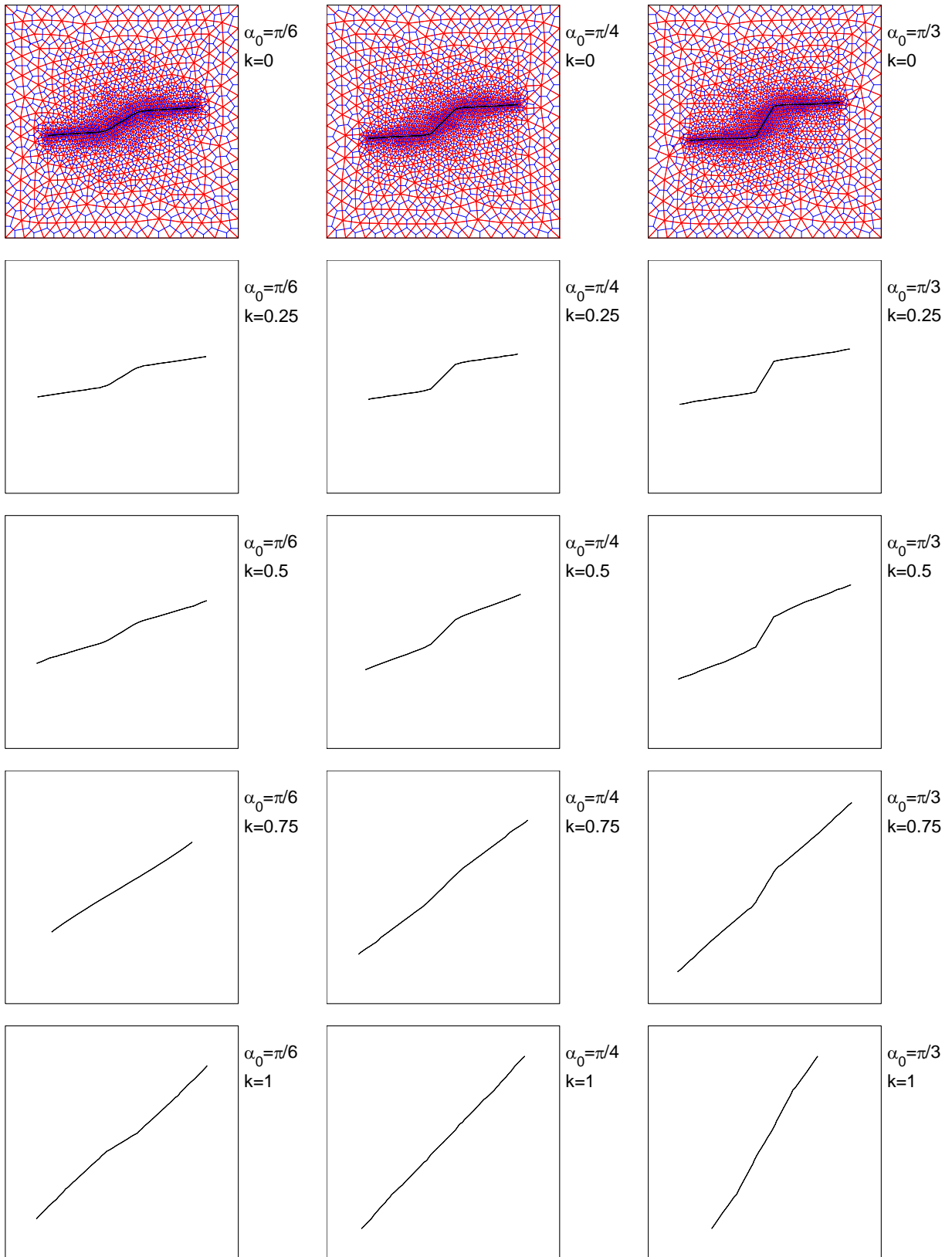
Figure 9: Crack trajectory for $\alpha_0 = \pi/6$, $\pi/4$, $\pi/3$, and $k = 0$, $1/4$, $1/2$, $3/4$, 1.

The Fig. 9 shows that the crack trajectory tends to approach an asymptote perpendicular to the external tensile load for all the three cases with $k = 0$. The asymptotic behavior is also evident for $k > 0$, but the asymptote is now oriented at an angle γ to the x-axis which depends on k:

$$\gamma = f(k). \tag{2}$$

The angle γ does not depend on the inclination α_0 of the initial straight crack, since the crack tends to propagate perpendicularly to the tensile principal direction of the uncracked plate.

For $k = 1$, the angle γ assumes the value $\pi/4$, in good accordance with the homogeneous state of stress represented by this load condition:

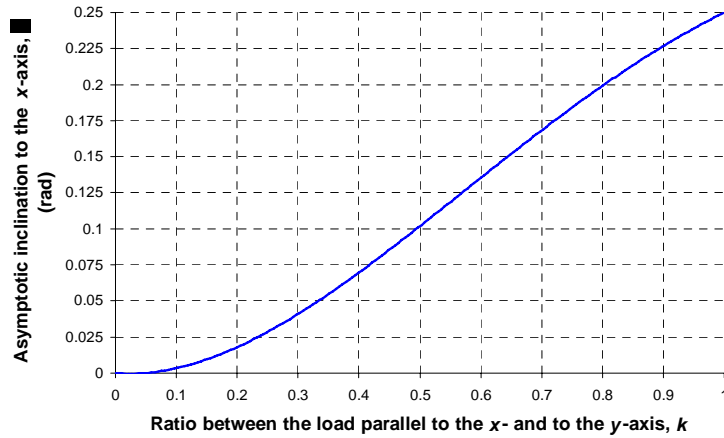$$\gamma\big|_{k=1} = \frac{\pi}{4}. \tag{3}$$

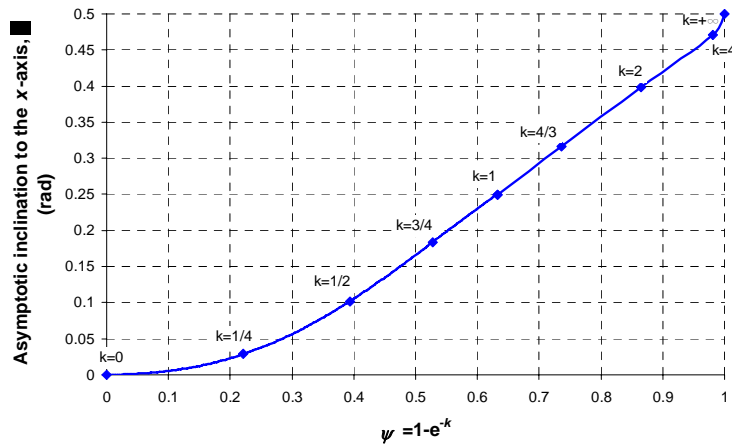Figure 10: Relation between the angle γ and the ratio k.

Figure 11: Relation between the angle γ and the variable ψ.

8

The plot of the function $\gamma = f(k)$ is provided in Fig. 10 for the range of values $0 \leq k \leq 1$.

If we know the behavior of the function $\gamma = f(k)$ for $0 \leq k \leq 1$, the behavior of $\gamma = f(k)$ for $1 < k \leq +\infty$ is also known. Actually, the asymptotes of the crack trajectory for a given k and its reciprocal value, $1/k$, are symmetric with respect to the bisector of the first quadrant in Fig. 1, $y = x$. That is to say, the value assumed by γ for a given k is equal to the complementary angle of γ for $1/k$:

$$\gamma\left(\frac{1}{k}\right) = \frac{\pi}{2} - \gamma(k). \tag{4}$$

The plot of γ for the complete range of values $0 \leq k \leq +\infty$ can be provided by means of the following change of variable (Fig. 11):

$$\psi = 1 - e^{-k}. \tag{5}$$

Finally, by way of illustration of the code potentialities, results regarding the computation on a geometry of finite dimensions are here presented. The computation on finite geometries allows one to avoid the quantitative discrepancy between the experimental results and those calculated via- stress intensity factors [9], due to the effect of the specimen boundaries on the stress field around the growing crack. In the case of a beam with two opposite pre-cracks submitted to skew-symmetric mixed-mode load in four-points, the numerical result was so accurate as to allow the complete description of the crack trajectory (Fig. 12). In Fig. 12, the darker blue color corresponds to the maximal compressive stress and the darker red color corresponds to the maximal tensile stress.
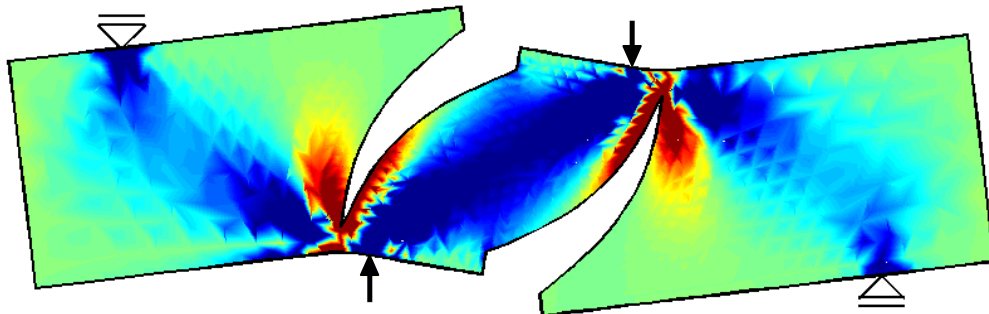

Figure 12: Displacement - stress analysis of the beam under skew-symmetric four point bending.

4 Conclusions

A first study on tensioned concrete plates was presented, based on an innovative size-insensitive constitutive law.
The numerical model adopted, based on the CM, allows analysis in the discrete. The crack propagation is then studied without using the stress intensity factors and without having to define a model to treat the zone ahead of the crack tip. This allows one to employ the same numerical code for bodies of different dimensions, geometries and boundary conditions, and for materials of different constitutive laws. As an example of the code versatility in front of the geometrical set-up, the interaction between two or more cracks oriented at any inclination and propagating in plate of finite/infinite dimensions

can be easily investigated. As regards the versatility in front of the constitutive law, the numerical analysis can be indifferently performed in the linear and non-linear field, with no adjunctive computational burdens.

The stress intensity factors of the variational approach can be estimated a-posteriori, since the code allows us to evaluate the compliance decrement following from a crack propagation. A comparison between the variational and discrete formulation is then possible, and it will be investigated in future studies.

In this paper, it was shown how the numerical results for plates in bi-axial loading are satisfactory with respect to the load and geometrical parameters.

The crack trajectory exhibits an asymptotic behavior which only depends on the ratio between the load parallel to the x- and the y-axis, k, being insensitive to the inclination α_0 of the initial straight crack.

The crack always tends to propagate perpendicularly to the tensile principal direction of the uncracked plate. The numerical law of the asymptotic inclination, said $\gamma = f(k)$, was provided for the complete variability range of k, $k = [0, +\infty]$.

The plausibility of the founded crack trajectories in dependence on the parameter k indirectly gives a validation of the adopted constitutive law for bi-axial tensile loading.

It was also shown how the numerical crack trajectory for a solid of finite dimensions is highly accurate.

Finally, it is remarkable how the analysis for finite solids is performed directly, without having to apply corrective factors to the solution on an infinite geometry in the same load conditions.

Acknowledgements

References

[1] D. S. Dugdale, *Yielding of Steel Sheets Containing Slits*, J. Mech. and Phys. Solids, 8(2), (1960), 100-104.

[2] E. Ferretti, *Crack propagation modeling by remeshing using the Cell Method (CM)*, to appear in CMES, editor-in-Chief Atluri (in press).

[3] E. Ferretti, *Identification of Constitutive Law in Mono-Axially Compressed Damaging Materials*, in M. Guagliano, M. H. Aliabadi, eds., *Advanced in Fracture and Damage Mechanics II. Proc. Int. Conf. on Fracture and Damage Mechanics,* FDM, Milano (2001), pp.379-384.

[4] A. A. Griffith, *The Phenomena of Rupture and Flow in Solids*, Philos. Trans. Roy. Soc. London. Ser. A, (1920), 221, 163-198.

[5] G. R. Irwin, Analysis of Stresses and Strains Near the Ends of a Crack Traversing a Plate, J. Appl. Mech., 24(3), (1957), 361-364.

[6] M. Y. Lenov, V. V. Panasyuk, *Development of Minute Cracks in Solids*, Prikl. Meckhanika, 5(4), (1959), 391-401.

[7] F. McClintock, *Ductile Fracture Instability in Shear*, J. Appl. Mech., 25, (1958), 582-588.

[8] V. V. Novozhilov, On the Foundations of the Theory of Equilibrium Cracks in Elastic Bodies, Prikl. Matem. i Mekhan., 33(5), (1969), 797-812.

[9] V. Z. Patron, E. M. Morozov, *Elastic-Plastic Fracture Mechanics*, MIR Publishers, (1978).

[10] E. Tonti, *A Finite Formulation of Field Equations: the Cell Method*, to appear in CMES, editor-in-Chief Atluri (in press).

[11] A. A. Wells, *Unstable Crack Propagation in Metals: Cleavage and Fast Fracture. Proc. Crack Propagation Symposium*, Vol. 1, College of Aeronautics and the Royal Aeronautical Society, Cranfield (England), (1961), pp. 210-230.