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Analysis of Mixed Mode Crack Initiation Angles for Orthotropic Material under Biaxial Loading

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Abstract

In the present paper fracture criteria for predicting crack initiation angles in an orthotropic homogeneous plate, with an inclined crack and subjected at infinity to a biaxial uniform load, are studied. The crack initiation angle q_0 can be calculated as a function of crack geometry and external loading applied at infinity. The numerical analysis is performed for a wide range of anisotropic material properties and applied loads.

Singular solution to specify elastic fields is generally incorrect and the effect of non-singular terms of the series expansion for the stress at the crack tip region is underlined. The estimation of the error associated with the singular terms representation is pointed out for some calculated parameters involved in the numerical analysis.

Stress and displacement components including non-singular terms are calculated making use of an unconventional approach to the derivation of the complex variable expressions of the elastic fields.

1 Introduction

Of concern in this paper is the study of the elastostatic fracture behaviour of an orthotropic plate with an inclined crack and subjected at infinity to a biaxial uniform load.
For homogeneous, isotropic and linearly elastic material, several fracture criteria for predicting initiation angles have been proposed. Among the mentioned criteria we remind the ones governed by circumferential stress s_q [1], and the strain energy density factor S [4]. An analysis of mixed mode crack initiation angles under different load condition is reported in [3] in the case of isotropic material.
The main purpose of this paper is the extension of the fracture criteria, listed above, to predict the crack initiation angle q_0 in orthotropic material.
A paper engaged in this topic and related to the present one is that of Ye and Ayari [6], in which mixed models of crack growth are evaluated for orthotropic solids with material symmetries.
Another aim of this paper is to point out the effect of load biaxiality on the near tip elastic field and on the angle of incipient crack propagation.
The influence of load biaxiality on fracture response of cracked isotropic bodies has been studied by Eftis et al. [7], Eftis and Subramonian [8], among others. Another paper engaged in this topic and related to the present one is that of Lim et al. [2] in which the effects of the load parallel to the crack direction on the asymptotic Mode-I elastic fields are pointed out and its influence on the angle of initial crack extension are illustrated.
Finally another purpose is to report a simple method in order to obtain the complex variable formulation of plane orthotropic elasticity, which is an adjustment to the static case of an approach previously used by Piva and Viola [5] in solving elastodynamic problems. The equilibrium equations of an orthotropic medium are reduced to a first order system involving a four dimensional vector. A similarity transformation is introduced to obtain a canonical form as a couple of independent Cauchy-Riemann systems.

2 Basic equation

Consider an orthotropic homogeneous continuum with the axes of elastic symmetry coinciding with rectangular coordinates axes x, y, z The displacement component along the z-axis, as well as all derivates with respect to the z-variable are assumed to vanish. The constitutive equations become:

$$s_x = C_{11}\frac{\partial u}{\partial x} + C_{12}\frac{\partial v}{\partial y} \tag{1}$$

$$s_y = C_{12}\frac{\partial u}{\partial x} + C_{22}\frac{\partial v}{\partial y} \tag{2}$$

$$t_{xy} = C_{66}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \tag{3}$$

where $u(x,y)$ e $v(x,y)$ are the x and y components of the displacement vector, respectively.
The parameters in (1)-(3) are related to the elastic constants according as a state of plane strain or plane stress is considered.
The equilibrium equations are:

$$\frac{\partial^2 u}{\partial x^2} + a\,\frac{\partial^2 u}{\partial y^2} + 2b\,\frac{\partial^2 v}{\partial x \partial y} = 0 \tag{4}$$

$$\frac{\partial^2 v}{\partial x^2} + a_1 \frac{\partial^2 v}{\partial y^2} + 2b_1 \frac{\partial^2 u}{\partial x \partial y} = 0 \tag{5}$$

with:

$$a = \frac{C_{66}}{C_{11}}, \quad 2b = \frac{C_{12} + C_{66}}{C_{11}}, \quad a_1 = \frac{C_{22}}{C_{66}}, \quad 2b_1 = \frac{C_{12} + C_{66}}{C_{66}} = \frac{2b}{a} \tag{6}$$

By introducing the four dimensional vector function:

$$\Phi(x,y) = \left(\Phi_1,\ \Phi_2,\ \Phi_3,\ \Phi_4\right)^{\mathrm{T}} \equiv \left(\frac{\partial u}{\partial x},\ \frac{\partial u}{\partial y},\ \frac{\partial v}{\partial x},\ \frac{\partial v}{\partial y}\right) \tag{7}$$

the equilibrium equations (1)-(3) can be written as

$$\frac{\partial \Phi}{\partial x} + \mathrm{Á}\,\frac{\partial \Phi}{\partial y} = 0 \tag{8}$$

where A is the following constant matrix:

$$\mathrm{Á} = \begin{pmatrix} 0 & a & 2b & 0 \\ -1 & 0 & 0 & 0 \\ 2b_1 & 0 & 0 & a_1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \tag{9}$$

Limiting attention to the imaginary eigenvalues of matrix A, one obtains:

$$l_1 = ip_1, \qquad l_2 = -ip_1, \qquad l_3 = ip_2, \qquad l_4 = -ip_2 \tag{10}$$

with

$$p_1 = \left(a_1 - \sqrt{a_1^{\,2} - a_2}\right)^{\frac{1}{2}}, \ p_2 = \left(a_1 + \sqrt{a_1^{\,2} - a_2}\right)^{\frac{1}{2}}, \ a_1 = \frac{a + a_1 - 4bb_1}{2}, \ a_2 = a\,a_1 \tag{11}$$

Turning to the approach already used in [5], equation (8) simplifies to:

$$\frac{\partial Y}{\partial x} + \mathrm{Â}\,\frac{\partial Y}{\partial y} = 0 \tag{12}$$

where the vector Y and the matrix B are linked to the vector Φ and matrix A by

$$Y\ (x,y) = U^{-1}\Phi(x,y)\ \ ,\ \ U = \begin{pmatrix} 0 & \dfrac{2b\,p_1^2}{a-p_1^2} & 0 & \dfrac{2b\,p_2^2}{a-p_2^2} \\ \dfrac{2b\,p_1}{a-p_1^2} & 0 & \dfrac{2b\,p_2}{a-p_2^2} & 0 \\ -p_1 & 0 & -p_2 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}\ \ ,\ \ \mathrm{B}=U^{-1}\mathrm{A}U \tag{13}$$

In terms of its components equation (12) gives two decoupled systems of the Cauchy-Riemann type which are satisfied by the following analytic functions:

$$\begin{aligned} W_1(z_1) &= y_1(x,y_1) + iy_2\,(x,y_1) \quad , \quad z_1 = x + iy_1 \quad , \quad y_1 = \frac{y}{p_1} \\ W_2(z_2) &= y_3(x,y_2) + iy_4\,(x,y_2) \quad , \quad z_2 = x + iy_2 \quad , \quad y_2 = \frac{y}{p_2} \end{aligned} \tag{14}$$

Introducing the potentials

$$\begin{aligned} L_1(z_1) &= -ip_1l_1(p_1 - p_2)W_1(z_1) \\ L_2(z_2) &= ip_2l_2(p_1 - p_2)W_2(z_2) \end{aligned} \tag{15}$$

with

$$l_1 = \frac{2b - a + p_1^2}{a - p_1^2} \quad , \quad l_2 = \frac{2b - a - p_2^2}{a - p_2^2} \tag{16}$$

and using (12), (15) and (16), equations (1)-(3) lead to the stress components as follows

$$s_x = \frac{C_{66}}{p_1p_2(p_2 - p_1)}\mathrm{Re}\left[p_1L_2(z_2) - p_2L_1(z_1)\right] \tag{17}$$

$$s_y = \frac{C_{66}}{p_2 - p_1}\mathrm{Re}\left[p_1L_1(z_1) - p_2L_2(z_2)\right] \tag{18}$$

$$t_{xy} = \frac{C_{66}}{p_2 - p_1}\mathrm{Im}\left[L_1(z_1) - L_2(z_2)\right] \tag{19}$$

From equations (13) we obtain the displacement components:

$$u(x,y) = \frac{2b}{p_1 - p_2}\mathrm{Re}\left[\frac{p_1}{l_1\left(a - p_1^2\right)}\,l_1(z_1) - \frac{p_2}{l_2\left(a - p_2^2\right)}\,l_2(z_2)\right] \tag{20}$$

$$v(x,y) = \frac{1}{p_1 - p_2}\mathrm{Im}\left[\frac{l_1(z_1)}{l_1} - \frac{l_2(z_2)}{l_2}\right] \tag{21}$$

Where $l_1(z_1)$ and $l_2(z_2)$ are the primitives of $L_1(z_1)$ and $L_2(z_2)$, respectively.
It can be shown that the uniform stress field at infinity

$$s_x^{(\infty)} = \mathrm{T}_1 \quad , \quad s_y^{(\infty)} = \mathrm{T}_2 \quad , \quad t_{xy}^{(\infty)} = \mathrm{T}_3 \tag{22}$$

leads to the following behaviour of the potential functions

$$L_1(z_1) = L_1^0 - \frac{p_1\left(p_2^2\mathrm{T}_1 + \mathrm{T}_2\right)}{C_{66}\left(p_1 + p_2\right)} + i\frac{\left(p_2 - p_1\right)}{2C_{66}}\mathrm{T}_3 \tag{23}$$

$$L_2(z_2) = L_2^0 - \frac{p_2\left(p_1^2\mathrm{T}_1 + \mathrm{T}_2\right)}{C_{66}\left(p_1 + p_2\right)} - i\frac{\left(p_2 - p_1\right)}{2C_{66}}\mathrm{T}_3 \tag{24}$$

where $L_1^0(z_1)$ and $L_2^0(z_2)$ are the analytic functions vanishing at infinity.

3 The problem of inclined crack

Consider an orthotropic and infinite medium with a crack , of length of 2*l*, inclined by an angle ω with respect to X-axis of a Cartesian orthogonal system O (X,Y) (fig. 1). The crack is supposed to be aligned with one of the three orthogonal axes of elastic symmetry of the body, coincident with *x*-axis of the coordinate system O (*x*,*y*).

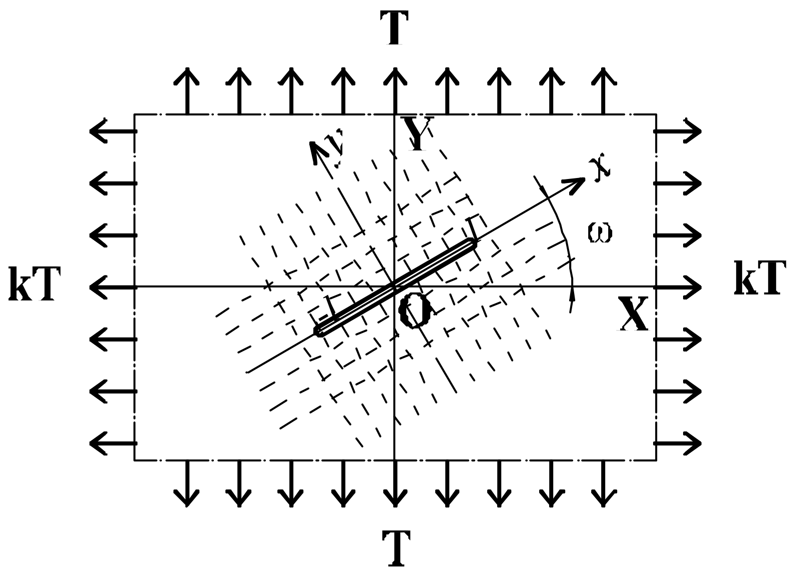

Fig. 1. The inclined crack geometry

We admit that the orthotropic body is subjected at infinity to a uniform biaxial load applied along X and Y directions. Referring to the orthogonal system O (*x*,*y*), the uniform biaxial load can be expressed by:

$$s_x^{(\infty)} \equiv T_1 = \frac{T}{2}\left[(1+k)-(1-k)\cos 2\omega\right] \tag{25}$$

$$s_y^{(\infty)} \equiv T_2 = \frac{T}{2}\left[(1+k)+(1-k)\cos 2\omega\right] \tag{26}$$

$$t_{xy}^{(\infty)} \equiv T_3 = \frac{T}{2}(1-k)\sin 2\omega \tag{27}$$

where k is the biaxial load factor. The problem is solved by determining the analytic functions $L_1^0(z_1)$ and $L_2^0(z_2)$ of equations (23) and (24), for Mode-I and Mode-II problems with uniform self-equilibrating tractions applied to the edges of the crack and null stress at infinity. Making use of expressions (17) – (19), the stress components can be found:

$$s_x = T_1 - \frac{T_2}{p_1 p_2} + \frac{1}{p_1 p_2 (p_2 - p_1)} \left\{ \begin{array}{l} T_2\left[p_2 \operatorname{Re} F_1(z_1) - p_1 \operatorname{Re} F_2(z_2)\right] + \\ + T_3\left[p_2^2 \operatorname{Im} F_1(z_1) - p_1^2 \operatorname{Im} F_2(z_2)\right] \end{array} \right\} \tag{28}$$

$$s_y = \frac{1}{p_2 - p_1}\left\{T_2\left[p_2 \operatorname{Re} F_2(z_2) - p_1 \operatorname{Re} F_1(z_1)\right] + p_1 p_2 T_3\left[\operatorname{Im} F_2(z_2) - \operatorname{Im} F_1(z_1)\right]\right\} \tag{29}$$

$$t_{xy} = \frac{1}{p_2 - p_1}\left\{T_3\left[p_2 \operatorname{Re} F_1(z_1) - p_1 \operatorname{Re} F_2(z_2)\right] + T_2\left[\operatorname{Im} F_2(z_2) - \operatorname{Im} F_1(z_1)\right]\right\} \tag{30}$$

as well as the displacement component:

$$\begin{aligned} u(x,y) = {} & \frac{2b}{C_{66}(p_1 - p_2)} \left\{ \begin{array}{l} T_2\left[\dfrac{p_2}{l_2(a-p_2^2)} \operatorname{Re} G_2(z_2) - \dfrac{p_1}{l_1(a-p_1^2)} \operatorname{Re} G_1(z_1)\right] + \\ + p_1 p_2 T_3\left[\dfrac{1}{l_2(a-p_2^2)} \operatorname{Im} G_2(z_2) - \dfrac{1}{l_1(a-p_1^2)} \operatorname{Im} G_1(z_1)\right] \end{array} \right\} - \\ & - \frac{2b\, p_1 p_2 (T_2 - p_1 p_2 T_1)}{C_{66} l_1 l_2 (a-p_1^2)(a-p_2^2)} x - \frac{b\,(p_1+p_2)^2\, T_3}{C_{66} l_1 l_2 (a-p_1^2)(a-p_2^2)} y \end{aligned} \tag{31}$$

$$\begin{aligned} v(x,y) = {} & \frac{1}{C_{66} l_1 l_2 (p_1 - p_2)} \left\{ \begin{array}{l} T_2\left[l_1 \operatorname{Im} G_2(z_2) - l_2 \operatorname{Im} G_1(z_1)\right] + \\ + T_3\left[l_2 p_2 \operatorname{Re} G_1(z_1) - l_1 p_1 \operatorname{Re} G_2(z_2)\right] \end{array} \right\} + \\ & + \frac{(T_2 - p_1 p_2 T_1)}{C_{66} l_1 l_2 (p_1^2 - p_2^2)} \left(\frac{l_2 p_2}{p_1} - \frac{l_1 p_1}{p_2}\right) y + \frac{(p_1 + p_2) T_3}{2 C_{66} l_1 l_2 (p_1 - p_2)} (l_1 - l_2) x \end{aligned} \tag{32}$$

with

$$F_j(z_j) = \frac{z_j}{\sqrt{z_j^2 - l^2}} \qquad G_j(z_j) = \sqrt{z_j^2 - l^2} \quad , \qquad j=1,2. \tag{33}$$

4 The near tip elastic fields

By use of Taylor series expansion of function $F(z_j)$ and $G(z_j)$, setting:

$$r_j = r g_j(q) \quad , \quad q_j = tg^{-1}\left(\frac{y}{p_j x}\right) = tg^{-1}\left(\frac{tgq}{p_j}\right) \quad , \; g_j(q) = \left(\cos^2 q + \frac{\sin^2 q}{p_j^{\,2}}\right)^{\frac{1}{2}} \tag{34}$$

And introducing the stress intensity factors $\mathrm{K}_{\mathrm{I}} = \mathrm{T}_2\sqrt{p\,l}$, $\mathrm{K}_{\mathrm{II}} = \mathrm{T}_3\sqrt{p\,l}$, the following asymptotic expressions of stress components can be obtained:

$$s_x = \frac{\mathrm{K}_{\mathrm{I}}}{\sqrt{p\,l}}\left(l - \frac{1}{p_1 p_2}\right) + \frac{(2pr)^{-\frac{1}{2}}}{p_1 p_2 (p_2 - p_1)}\left\{\mathrm{K}_{\mathrm{I}}\left[p_2\frac{\cos\frac{q_1}{2}}{\sqrt{g_1(q)}} - p_1\frac{\cos\frac{q_2}{2}}{\sqrt{g_2(q)}}\right] + \mathrm{K}_{\mathrm{II}}\left[p_1^2\frac{\sin\frac{q_2}{2}}{\sqrt{g_2(q)}} - p_2^2\frac{\sin\frac{q_1}{2}}{\sqrt{g_1(q)}}\right]\right\} \tag{35}$$

$$s_y = \frac{1}{(p_2 - p_1)}\sqrt{\frac{1}{2pr}}\left\{\mathrm{K}_{\mathrm{I}}\left[p_2\frac{\cos\frac{q_2}{2}}{\sqrt{g_2(q)}} - p_1\frac{\cos\frac{q_1}{2}}{\sqrt{g_1(q)}}\right] + p_1 p_2 \mathrm{K}_{\mathrm{II}}\left[\frac{\sin\frac{q_1}{2}}{\sqrt{g_1(q)}} - \frac{\sin\frac{q_2}{2}}{\sqrt{g_2(q)}}\right]\right\} \tag{36}$$

$$t_{xy} = \frac{1}{(p_2 - p_1)}\sqrt{\frac{1}{2p\,r}}\left\{\mathrm{K}_{\mathrm{II}}\left[p_2\frac{\cos\frac{q_1}{2}}{\sqrt{g_1(q)}} - p_1\frac{\cos\frac{q_2}{2}}{\sqrt{g_2(q)}}\right] + \mathrm{K}_{\mathrm{I}}\left[\frac{\sin\frac{q_1}{2}}{\sqrt{g_1(q)}} - \frac{\sin\frac{q_2}{2}}{\sqrt{g_2(q)}}\right]\right\} \tag{37}$$

Similarly for displacement components:

$$\begin{aligned} u = {} & \frac{2b}{C_{66}(p_1 - p_2)}\sqrt{\frac{2r}{p}}\left\{\begin{aligned} & \mathrm{K}_{\mathrm{I}}\left[\frac{p_2\sqrt{g_2(q)}}{l_2(a - p_2^2)}\cos\frac{q_2}{2} - \frac{p_1\sqrt{g_1(q)}}{l_1(a - p_1^2)}\cos\frac{q_1}{2}\right] + \\ & + p_1 p_2 \mathrm{K}_{\mathrm{II}}\left[\frac{\sqrt{g_2(q)}}{l_2(a - p_2^2)}\sin\frac{q_2}{2} - \frac{\sqrt{g_1(q)}}{l_1(a - p_1^2)}\sin\frac{q_1}{2}\right]\end{aligned}\right\} - \\ & - \frac{2b\,p_1 p_2(\mathrm{K}_{\mathrm{I}} - p_1 p_2 l\,\mathrm{K}_{\mathrm{I}})}{\sqrt{p\,l}\,C_{66} l_1 l_2 (a - p_1^2)(a - p_2^2)}(l + r\cos q) - \frac{b\,\mathrm{K}_{\mathrm{II}}(p_1 + p_2)^2}{C_{66} l_1 l_2 (a - p_1^2)(a - p_2^2)}\frac{r\sin q}{\sqrt{p\,l}} \end{aligned} \tag{38}$$

$$\begin{aligned} v = {} & \frac{1}{C_{66}(p_1 - p_2)}\frac{\sqrt{2r}}{\sqrt{p}\,l_1 l_2}\left\{\begin{aligned} & \mathrm{K}_{\mathrm{I}}\left[l_1\sqrt{g_2(q)}\sin\frac{q_2}{2} - l_2\sqrt{g_1(q)}\sin\frac{q_1}{2}\right] + \\ & + \mathrm{K}_{\mathrm{II}}\left[l_2 p_2\sqrt{g_1(q)}\cos\frac{q_1}{2} - l_1 p_1\sqrt{g_1(q)}\cos\frac{q_2}{2}\right]\end{aligned}\right\} + \\ & + \frac{\mathrm{K}_{\mathrm{II}}(p_1 + p_2)(l_1 - l_2)}{2C_{66} l_1 l_2 (p_1 - p_2)}\frac{(l + r\cos q)}{\sqrt{p\,l}} + \frac{(\mathrm{K}_{\mathrm{I}} - p_1 p_2 l\,\mathrm{K}_{\mathrm{I}})}{C_{66}(p_1^2 - p_2^2)}\left(\frac{p_2}{l_1 p_1} - \frac{p_1}{l_2 p_2}\right)\frac{r\sin q}{\sqrt{p\,l}} \end{aligned} \tag{39}$$

5 Fracture criteria

5.1 Maximum circumferential tensile stress theory

The maximum circumferential tensile stress criterion for isotropic materials maintains that the crack extension angle q_0 is individuated by the direction orthogonal to the one on which the circumferential stress s_q attains a positive maximum value. This criterion can not be applied to orthotropic materials for which the critical stress intensity factor varies with the polar angle q. Assuming that K_{IC}^x and K_{IC}^y are the critical stress intensity factors along the axes x and y of elastic symmetry, the critical stress intensity factor on the q plane will be:

$$K_{IC}^{q} = K_{IC}^{x} \sin^2 q + K_{IC}^{y} \cos^2 q \tag{40}$$

So, for orthotropic materials the maximum circumferential stress criterion [9] consists of finding the maximum of the following function:

$$R_q = \frac{s_q \sqrt{2 p r}}{K_{IC}^{q}} \tag{41}$$

From the numerical analysis point of view it is convenient to analyse the maximum of the normalised form of equation (41), which is:

$$\frac{s_q^*}{K_{IC}^{q} / K_{IC}^{x}} = \frac{\frac{s_q}{T}\sqrt{\frac{2r}{l}}}{K_{IC}^{q} / K_{IC}^{x}} = \frac{s_x \sin^2 q + s_y \cos^2 q - t_{xy} \sin 2q}{\sin^2 q + \frac{K_{IC}^{y}}{K_{IC}^{x}} \cos^2 q} \tag{42}$$

Considering three orthotropic materials, Glass-epoxy, Graphite-epoxy (V_f=0.65) and Graphite-epoxy (V_f=0.50) [10], equation (42) can be used to obtain the crack initiation angle q_0 for different load conditions. Fig. 2 shows the crack extension angle for the orthotropic materials mentioned, assuming that the ratio K_{IC}^x / K_{IC}^y is equal to the ratio of the elastic moduli along the x and y direction.
It can be noted that the crack extension angle, for a fixed value of crack extension angle, depends on elastic properties of the orthotropic material. When the difference between the elastic moduli on the principal direction becomes large, the crack extends with a small polar angle, meaning that the crack extends very near the collinear direction.

5.2 Strain energy density theory

The strain energy density criterion can be applied to predict the crack propagation in isotropic materials [4]. In this work the criterion is extended to an orthotropic medium with an inclined crack aligned with one of the axes of elastic symmetry of the body.
Making use of the stress and strain components s_{ij} and e_{ij}, the strain energy density can be written as

$$\frac{dW}{dV} = \frac{1}{2} s_{ij} e_{ij} \tag{43}$$

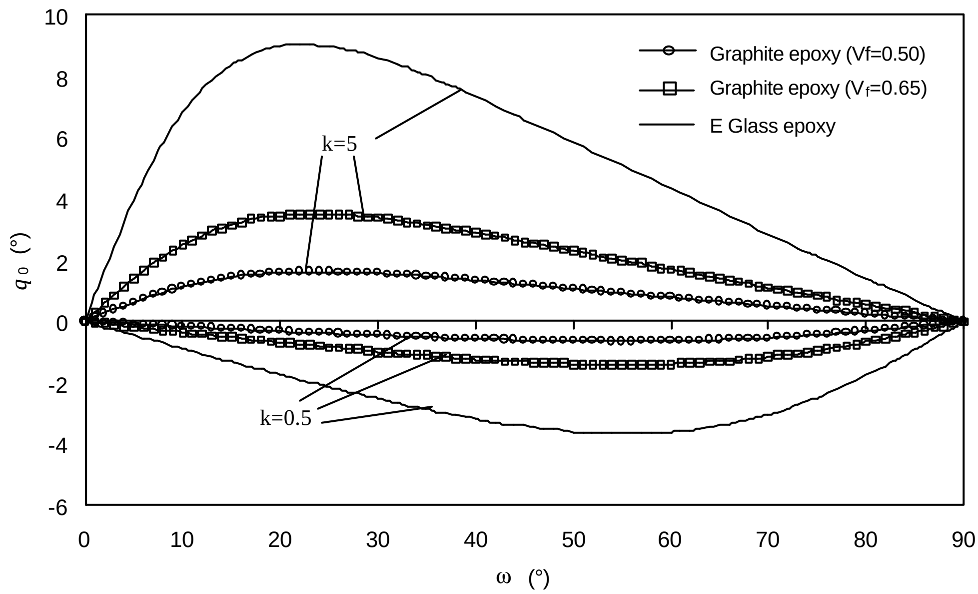

Fig. 2 Crack initiation angle vs. crack inclination angle for different orthotropic materials and for some values of the biaxial load parameter (using circumferential stress criterion).

For the elastostatic plane problem of an infinite orthotropic medium equation (43) becomes:

$$\frac{\mathrm{d}W}{\mathrm{d}V}=\frac{1}{2}\left[\frac{s_x^2 C_{22}+s_y^2 C_{11}-2s_x s_y C_{12}}{C_{11}C_{22}-C^2_{12}}+\frac{t^2_{xy}}{C_{66}}\right] \tag{44}$$

where the parameters that appear in equation (44) are related to elastic constants according as the case of plane stress or plane strain is considered.
For isotropic materials the relative minimum of dW/dV is assumed to be associated with the direction of crack initiation and the crack is assumed to grow when dW/dV reaches a critical value (dW/dV)$_c$.
In the orthotropic case, (dW/dV)$_c$ is a function of the polar angle which we admit to be as follows:

$$\left(\frac{\mathrm{d}W}{\mathrm{d}V}\right)_c^q=\left(\frac{\mathrm{d}W}{\mathrm{d}V}\right)_c^x \sin^2 q+\left(\frac{\mathrm{d}W}{\mathrm{d}V}\right)_c^y \cos^2 q \tag{45}$$

where (dW/dV)$_c^x$ and (dW/dV)$_c^y$ are the critical energy densities in x and y directions, respectively.
The crack initiation angle, for orthotropic materials, can be obtained minimizing the ratio (dW/dV)/(dW/dV)$_c^q$
Referring again to the Glass-epoxy, Graphite-epoxy (V_f=0.65) and Graphite-epoxy (V_f=0.50) [10], with a given $\left(\frac{\mathrm{d}W}{\mathrm{d}V}\right)_c^x \Big/ \left(\frac{\mathrm{d}W}{\mathrm{d}V}\right)_c^y$ ratio assumed to be equal to the square of elastic moduli ratio along the axes of elastic symmetry, the corresponding crack initiation angle q_0 can be obtained as a function of the crack inclination angle ω, for different values of the biaxial load parameter k (Fig. 3).
Note that similarly to maximum tensile stress criterion, the crack initiation angle depends on the elastic properties of the orthotropic material. In particular, for Glass-Epoxy q_0 is different form zero also for ω=0° and ω=90°.

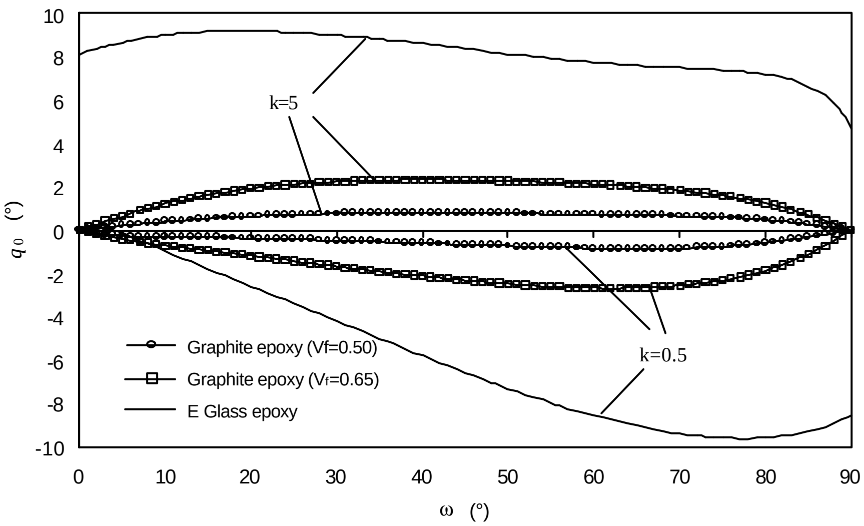

Fig. 3 Crack initiation angle vs. crack inclination angle for different orthotropic materials and for some values of the biaxial load parameter (strain energy density criterion).

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